DWaveNetworkX Documentation

Release 0.8.13

D-Wave Systems Inc

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Contents

D-Wave NetworkX is an extension of NetworkX—a Python language package for exploration and analysis of networks and network algorithms—for users of D-Wave Systems. It provides tools for working with Chimera graphs and implementations of graph-theory algorithms on the D-Wave system and other binary quadratic model samplers.

The example below generates a graph for a Chimera unit cell (eight nodes in a 4-by-2 bipartite architecture).

```
>>> import dwave_networkx as dnx
>>> graph = dnx.chimera_graph(1, 1, 4)
```

See the documentation for more examples.

CHAPTER 1

Installation

Installation from PyPi:

```
pip install dwave_networkx
```

Installation from source:

```
pip install -r requirements.txt
python setup.py install
```

CHAPTER 2

License

Released under the Apache License 2.0.

CHAPTER 3

Contributing

Ocean's contributing guide has guidelines for contributing to Ocean packages.

3.1 Documentation

Note: This documentation is for the latest version of dwave-networkx. Documentation for the version currently installed by dwave-ocean-sdk is here: dwave-networkx.

3.1.1 Introduction

D-Wave NetworkX provides tools for working with Chimera and Pegasus graphs and implementations of graph-theory algorithms on the D-Wave system and other binary quadratic model samplers; for example, functions such as *draw_chimera()* provide easy visualization for Chimera graphs; functions such as *maximum_cut()* or *min_vertex_cover()* provide graph algorithms useful to optimization problems that fit well with the D-Wave system.

Like the D-Wave system, all other supported samplers must have *sample_qubo* and *sample_ising* methods for solving Ising and QUBO models and return an iterable of samples in order of increasing energy. You can set a default sampler using the *set_default_sampler()* function.

- For an introduction to quantum processing unit (QPU) topologies such as the Chimera‘ and Pegasus graphs, see Topology.
- For an introduction to binary quadratic models (BQMs), see Binary Quadratic Models.
- For an introduction to samplers, see Samplers and Composites.

Example

Below you can see how to create Chimera graphs implemented in the D-Wave 2X and D-Wave 2000Q systems:

```
import dwave_networkx as dnx

# D-Wave 2X
C = dnx.chimera_graph(12, 12, 4)

# D-Wave 2000Q
C = dnx.chimera_graph(16, 16, 4)
```

3.1.2 Reference Documentation

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Algorithms

Implementations of graph-theory algorithms on the D-Wave system and other binary quadratic model samplers.

Canonicalization

canonical_chimera_labeling(G[, t])	Returns a mapping from the labels of G to chimera-indexed labeling.

dwave_networkx.canonical_chimera_labeling

canonical_chimera_labeling(*G*, *t=None*)

Returns a mapping from the labels of G to chimera-indexed labeling.

Parameters

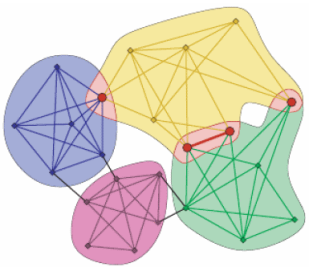
- **G** (*NetworkX graph*) – A Chimera-structured graph.
- **t** (*int (optional, default 4)*) – Size of the shore within each Chimera tile.

Returns **chimera_indices** – A mapping from the current labels to a 4-tuple of Chimera indices.

Return type dict

Clique

A clique in an undirected graph G = (V, E) is a subset of the vertex set such that for every two vertices in C there exists an edge connecting the two.



maximum_clique(G[, sampler, lagrange])	Returns an approximate maximum clique.
clique_number(G[, sampler, lagrange])	Returns the number of vertices in the maximum clique of a graph.
is_clique(G, clique_nodes)	Determines whether the given nodes form a clique.

dwave_networkx.maximum_clique

maximum_clique(*G*, *sampler=None*, *lagrange=2.0*, ***sampler_args*)

Returns an approximate maximum clique. A clique in an undirected graph, G = (V, E), is a subset of the vertex set $C \subseteq V$ such that for every two vertices in C there exists an edge connecting the two. This is equivalent to saying that the subgraph induced by C is complete (in some cases, the term clique may also refer to the subgraph). A maximum clique is a clique of the largest possible size in a given graph.

This function works by finding the maximum independent set of the compliment graph of the given graph G which is equivalent to finding maximum clique. It defines a QUBO with ground states corresponding to a maximum weighted independent set and uses the sampler to sample from it.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a maximum clique.
- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **lagrange** (*optional (default 2)*) – Lagrange parameter to weight constraints (no edges within set) versus objective (largest set possible).
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **clique_nodes** – List of nodes that form a maximum clique, as determined by the given sampler.

Return type list

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

References

Maximum Clique on Wikipedia

Independent Set on Wikipedia

QUBO on Wikipedia

dwave_networkx.clique_number

clique_number(*G*, *sampler=None*, *lagrange=2.0*, ***sampler_args*)

Returns the number of vertices in the maximum clique of a graph. A maximum clique is a clique of the largest possible size in a given graph. The clique number math:*omega(G)* of a graph G is the number of vertices in a maximum clique in G. The intersection number of G is the smallest number of cliques that together cover all edges of G.

This function works by finding the maximum independent set of the compliment graph of the given graph G which is equivalent to finding maximum clique. It defines a QUBO with ground states corresponding to a maximum weighted independent set and uses the sampler to sample from it.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a maximum clique.
- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **lagrange** (*optional (default 2)*) – Lagrange parameter to weight constraints (no edges within set) versus objective (largest set possible).
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **clique_nodes** – List of nodes that form a maximum clique, as determined by the given sampler.

Return type list

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

References

Maximum Clique on Wikipedia

dwave_networkx.is_clique

is_clique(*G*, *clique_nodes*)

Determines whether the given nodes form a clique.

A clique is a subset of nodes of an undirected graph such that every two distinct nodes in the clique are adjacent.

Parameters

- **G** (*NetworkX graph*) – The graph on which to check the clique nodes.
- **clique_nodes** (*list*) – List of nodes that form a clique, as determined by the given sampler.

Returns **is_clique** – True if clique_nodes forms a clique.

Return type bool

Example

This example checks two sets of nodes, both derived from a single Chimera unit cell, for an independent set. The first set is the horizontal tile's nodes; the second has nodes from the horizontal and verical tiles.

```
>>> import dwave_networkx as dnx
>>> G = dnx.chimera_graph(1, 1, 4)
>>> dnx.is_clique(G, [0, 1, 2, 3])
False
>>> dnx.is_clique(G, [0, 4])
True
```

Coloring

Graph coloring is the problem of assigning a color to the vertices of a graph in a way that no adjacent vertices have the same color.

Example

The map-coloring problem is to assign a color to each region of a map (represented by a vertex on a graph) such that any two regions sharing a border (represented by an edge of the graph) have different colors.

Fig. 1: Coloring a map of Canada with four colors.

is_vertex_coloring(G, coloring)	Determines whether the given coloring is a vertex coloring of graph G.
min_vertex_color(G[, sampler, chromatic_lb, ...])	Returns an approximate minimum vertex coloring.
min_vertex_color_qubo(G[, chromatic_lb, ...])	Return a QUBO with ground states corresponding to a minimum vertex coloring.
vertex_color(G, colors[, sampler])	Returns an approximate vertex coloring.
vertex_color_qubo(G, colors)	Return the QUBO with ground states corresponding to a vertex coloring.

dwave_networkx.algorithms.coloring.is_vertex_coloring

is_vertex_coloring(*G*, *coloring*)

Determines whether the given coloring is a vertex coloring of graph G.

Parameters

- **G** (*NetworkX graph*) – The graph on which the vertex coloring is applied.
- **coloring** (*dict*) – A coloring of the nodes of G. Should be a dict of the form {node: color, ...}.

Returns **is_vertex_coloring** – True if the given coloring defines a vertex coloring; that is, no two adjacent vertices share a color.

Return type bool

Example

This example colors checks two colorings for a graph, G, of a single Chimera unit cell. The first uses one color (0) for the four horizontal qubits and another (1) for the four vertical qubits, in which case there are no adjacencies; the second coloring swaps the color of one node.

```
>>> G = dnx.chimera_graph(1,1,4)
>>> colors = {0: 0, 1: 0, 2: 0, 3: 0, 4: 1, 5: 1, 6: 1, 7: 1}
>>> dnx.is_vertex_coloring(G, colors)
True
>>> colors[4]=0
>>> dnx.is_vertex_coloring(G, colors)
False
```

dwave_networkx.algorithms.coloring.min_vertex_color

min_vertex_color(*G*, *sampler=None*, *chromatic_lb=None*, *chromatic_ub=None*, ***sampler_args*)

Returns an approximate minimum vertex coloring.

Vertex coloring is the problem of assigning a color to the vertices of a graph in a way that no adjacent vertices have the same color. A minimum vertex coloring is the problem of solving the vertex coloring problem using the smallest number of colors.

Defines a QUBO [DWMP] with ground states corresponding to minimum vertex colorings and uses the sampler to sample from it.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a minimum vertex coloring.
- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **chromatic_lb** (*int, optional*) – A lower bound on the chromatic number. If one is not provided, a bound is calulcated.

- **chromatic_ub** (*int, optional*) – An upper bound on the chromatic number. If one is not provided, a bound is calculated.
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **coloring** – A coloring for each vertex in G such that no adjacent nodes share the same color. A dict of the form {node: color, ...}

Return type dict

References

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

dwave_networkx.algorithms.coloring.min_vertex_color_qubo

min_vertex_color_qubo(*G*, *chromatic_lb=None*, *chromatic_ub=None*)

Return a QUBO with ground states corresponding to a minimum vertex coloring.

Vertex coloring is the problem of assigning a color to the vertices of a graph in a way that no adjacent vertices have the same color. A minimum vertex coloring is the problem of solving the vertex coloring problem using the smallest number of colors.

Defines a QUBO [DWMP] with ground states corresponding to minimum vertex colorings and uses the sampler to sample from it.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a minimum vertex coloring.
- **chromatic_lb** (*int, optional*) – A lower bound on the chromatic number. If one is not provided, a bound is calulcated.
- **chromatic_ub** (*int, optional*) – An upper bound on the chromatic number. If one is not provided, a bound is calculated.
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **QUBO** – The QUBO with ground states corresponding to minimum colorings of the graph. The QUBO variables are labelled *(v, c)* where *v* is a node in *G* and *c* is a color. In the ground state of the QUBO, a variable *(v, c)* has value 1 if *v* should be colored *c* in a valid coloring.

Return type dict

dwave_networkx.algorithms.coloring.vertex_color

vertex_color(*G*, *colors*, *sampler=None*, ***sampler_args*)

Returns an approximate vertex coloring.

Vertex coloring is the problem of assigning a color to the vertices of a graph in a way that no adjacent vertices have the same color.

Defines a QUBO [DWMP] with ground states corresponding to valid vertex colorings and uses the sampler to sample from it.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a minimum vertex coloring.
- **colors** (*int/sequence*) – The colors. If an int, the colors are labelled *[0, n)*. The number of colors must be greater or equal to the chromatic number of the graph.
- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **coloring** – A coloring for each vertex in G such that no adjacent nodes share the same color. A dict of the form {node: color, ... }

Return type dict

References

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

dwave_networkx.algorithms.coloring.vertex_color_qubo

vertex_color_qubo(*G*, *colors*)

Return the QUBO with ground states corresponding to a vertex coloring.

If V is the set of nodes, E is the set of edges and C is the set of colors the resulting qubo will have:

- $|V| * |C|$ variables/nodes
- $|V| * |C| * (|C| - 1)/2 + |E| * |C|$ interactions/edges

The QUBO has ground energy $-|V|$ and an infeasible gap of 1.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a minimum vertex coloring.
- **colors** (*int/sequence*) – The colors. If an int, the colors are labelled *[0, n)*. The number of colors must be greater or equal to the chromatic number of the graph.

Returns **QUBO** – The QUBO with ground states corresponding to valid colorings of the graph. The QUBO variables are labelled *(v, c)* where *v* is a node in *G* and *c* is a color. In the ground state of the QUBO, a variable *(v, c)* has value 1 if *v* should be colored *c* in a valid coloring.

Return type dict

Cover

Vertex covering is the problem of finding a set of vertices such that all the edges of the graph are incident to at least one of the vertices in the set.

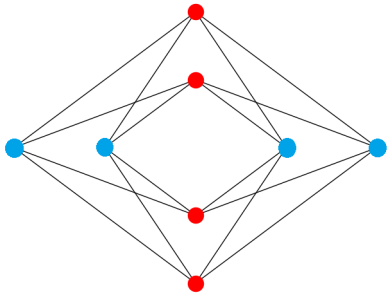


Fig. 2: Cover for a Chimera unit cell: the nodes of both the blue set of vertices (the horizontal tile of the Chimera unit cell) and the red set (vertical tile) connect to all 16 edges of the graph.

min_weighted_vertex_cover(G[, weight, ...])	Returns an approximate minimum weighted vertex cover.
min_vertex_cover(G[, sampler, lagrange])	Returns an approximate minimum vertex cover.
is_vertex_cover(G, vertex_cover)	Determines whether the given set of vertices is a vertex cover of graph G.

dwave_networkx.algorithms.cover.min_weighted_vertex_cover

min_weighted_vertex_cover(*G*, *weight=None*, *sampler=None*, *lagrange=2.0*, ***sampler_args*)

Returns an approximate minimum weighted vertex cover.

Defines a QUBO with ground states corresponding to a minimum weighted vertex cover and uses the sampler to sample from it.

A vertex cover is a set of vertices such that each edge of the graph is incident with at least one vertex in the set. A minimum weighted vertex cover is the vertex cover of minimum total node weight.

Parameters

- **G** (*NetworkX graph*) –
- **weight** (*string, optional (default None)*) – If None, every node has equal weight. If a string, use this node attribute as the node weight. A node without this attribute is assumed to have max weight.
- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **lagrange** (*optional (default 2)*) – Lagrange parameter to weight constraints versus objective.
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **vertex_cover** – List of nodes that the form a the minimum weighted vertex cover, as determined by the given sampler.

Return type list

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

https://en.wikipedia.org/wiki/Vertex_cover

https://en.wikipedia.org/wiki/Quadratic_unconstrained_binary_optimization

dwave_networkx.algorithms.cover.min_vertex_cover

min_vertex_cover(*G*, *sampler=None*, *lagrange=2.0*, ***sampler_args*)

Returns an approximate minimum vertex cover.

Defines a QUBO with ground states corresponding to a minimum vertex cover and uses the sampler to sample from it.

A vertex cover is a set of vertices such that each edge of the graph is incident with at least one vertex in the set. A minimum vertex cover is the vertex cover of smallest size.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a minimum vertex cover.
- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **lagrange** (*optional (default 2)*) – Lagrange parameter to weight constraints versus objective.
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **vertex_cover** – List of nodes that form a minimum vertex cover, as determined by the given sampler.

Return type list

Examples

This example uses a sampler from dimod to find a minimum vertex cover for a Chimera unit cell. Both the horizontal (vertices 0,1,2,3) and vertical (vertices 4,5,6,7) tiles connect to all 16 edges, so repeated executions can return either set.

```
>>> import dwave_networkx as dnx
>>> import dimod
>>> sampler = dimod.ExactSolver()  # small testing sampler
>>> G = dnx.chimera_graph(1, 1, 4)
>>> G.remove_node(7)  # to give a unique solution
>>> dnx.min_vertex_cover(G, sampler, lagrange=2.0)
[4, 5, 6]
```

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

References

https://en.wikipedia.org/wiki/Vertex_cover

https://en.wikipedia.org/wiki/Quadratic_unconstrained_binary_optimization

dwave_networkx.algorithms.cover.is_vertex_cover

is_vertex_cover(*G*, *vertex_cover*)

Determines whether the given set of vertices is a vertex cover of graph G.

A vertex cover is a set of vertices such that each edge of the graph is incident with at least one vertex in the set.

Parameters

- **G** (*NetworkX graph*) – The graph on which to check the vertex cover.
- **vertex_cover** – Iterable of nodes.

Returns **is_cover** – True if the given iterable forms a vertex cover.

Return type bool

Examples

This example checks two covers for a graph, G, of a single Chimera unit cell. The first uses the set of the four horizontal qubits, which do constitute a cover; the second set removes one node.

```
>>> import dwave_networkx as dnx
>>> G = dnx.chimera_graph(1, 1, 4)
>>> cover = [0, 1, 2, 3]
>>> dnx.is_vertex_cover(G,cover)
True
>>> cover = [0, 1, 2]
>>> dnx.is_vertex_cover(G,cover)
False
```

Elimination Ordering

Many algorithms for NP-hard problems are exponential in treewidth. However, finding a lower bound on treewidth is in itself NP-complete. [GD] describes a branch-and-bound algorithm for computing the treewidth of an undirected graph by searching in the space of *perfect elimination ordering* of vertices of the graph.

A *clique* of a graph is a fully-connected subset of vertices; that is, every pair of vertices in the clique share an edge. A *simplicial* vertex is one whose neighborhood induces a clique. A perfect elimination ordering is an ordering of vertices $1..n$ such that any vertex i is simplicial for the subset of vertices $i..n$.

chimera_elimination_order(m[, n, t, coordinates])	Provides a variable elimination order for a Chimera graph.
elimination_order_width(G, order)	Calculates the width of the tree decomposition induced by a variable elimination order.
is_almost_simplicial(G, n)	Determines whether a node n in G is almost simplicial.
is_simplicial(G, n)	Determines whether a node n in G is simplicial.
max_cardinality_heuristic(G)	Computes an upper bound on the treewidth of graph G based on the max-cardinality heuristic for the elimination ordering.
minor_min_width(G)	Computes a lower bound for the treewidth of graph G.
min_fill_heuristic(G)	Computes an upper bound on the treewidth of graph G based on the min-fill heuristic for the elimination ordering.

Continued on next page

Table 5 – continued from previous page

min_width_heuristic(G)	Computes an upper bound on the treewidth of graph G based on the min-width heuristic for the elimination ordering.
pegasus_elimination_order(n[, coordinates])	Provides a variable elimination order for the Pegasus graph.
treewidth_branch_and_bound(G[, ...])	Computes the treewidth of graph G and a corresponding perfect elimination ordering.

dwave_networkx.algorithms.elimination_ordering.chimera_elimination_order

chimera_elimination_order(*m*, *n=None*, *t=4*, *coordinates=False*)

Provides a variable elimination order for a Chimera graph.

A graph defined by `chimera_graph(m,n,t)` has treewidth $max(m, n) * t$. This function outputs a variable elimination order inducing a tree decomposition of that width.

Parameters

- **m** (*int*) – Number of rows in the Chimera lattice.
- **n** (*int (optional, default m)*) – Number of columns in the Chimera lattice.
- **t** (*int (optional, default 4)*) – Size of the shore within each Chimera tile.
- **bool (optional, default False)** (*coordinates*) – If True, the elimination order is given in terms of 4-term Chimera coordinates, otherwise given in linear indices.

Returns **order** – An elimination order that induces the treewidth of chimera_graph(m,n,t).

Return type list

Examples

```
>>> G = dnx.chimera_elimination_order(1, 1, 4)  # a single Chimera tile
```

dwave_networkx.algorithms.elimination_ordering.elimination_order_width

elimination_order_width(*G*, *order*)

Calculates the width of the tree decomposition induced by a variable elimination order.

Parameters

- **G** (*NetworkX graph*) – The graph on which to compute the width of the tree decomposition.
- **order** (*list*) – The elimination order. Must be a list of all of the variables in G.

Returns **treewidth** – The width of the tree decomposition induced by order.

Return type int

Examples

This example computes the width of the tree decomposition for the K_4 complete graph induced by an elimination order found through the min-width heuristic.

```
>>> K_4 = nx.complete_graph(4)
>>> tw, order = dnx.min_width_heuristic(K_4)
>>> print(tw)
3
>>> dnx.elimination_order_width(K_4, order)
3
```

dwave_networkx.algorithms.elimination_ordering.is_almost_simplicial

is_almost_simplicial(*G*, *n*)

Determines whether a node n in G is almost simplicial.

Parameters

- **G** (*NetworkX graph*) – The graph on which to check whether node n is almost simplicial.
- **n** (*node*) – A node in graph G.

Returns **is_almost_simplicial** – True if all but one of its neighbors induce a clique

Return type bool

Examples

This example checks whether node 0 is simplicial or almost simplicial for a K_5 complete graph with one edge removed.

```
>>> K_5 = nx.complete_graph(5)
>>> K_5.remove_edge(1,3)
>>> dnx.is_simplicial(K_5, 0)
False
>>> dnx.is_almost_simplicial(K_5, 0)
True
```

dwave_networkx.algorithms.elimination_ordering.is_simplicial

is_simplicial(*G*, *n*)

Determines whether a node n in G is simplicial.

Parameters

- **G** (*NetworkX graph*) – The graph on which to check whether node n is simplicial.
- **n** (*node*) – A node in graph G.

Returns **is_simplicial** – True if its neighbors form a clique.

Return type bool

Examples

This example checks whether node 0 is simplicial for two graphs: G, a single Chimera unit cell, which is bipartite, and K_5, the K_5 complete graph.

```
>>> G = dnx.chimera_graph(1, 1, 4)
>>> K_5 = nx.complete_graph(5)
>>> dnx.is_simplicial(G, 0)
False
>>> dnx.is_simplicial(K_5, 0)
True
```

dwave_networkx.algorithms.elimination_ordering.max_cardinality_heuristic

max_cardinality_heuristic(*G*)

Computes an upper bound on the treewidth of graph G based on the max-cardinality heuristic for the elimination ordering.

Parameters **G** (*NetworkX graph*) – The graph on which to compute an upper bound for the treewidth.

Returns

- **treewidth_upper_bound** (*int*) – An upper bound on the treewidth of the graph G.
- **order** (*list*) – An elimination order that induces the treewidth.

Examples

This example computes an upper bound for the treewidth of the K_4 complete graph.

```
>>> K_4 = nx.complete_graph(4)
>>> tw, order = dnx.max_cardinality_heuristic(K_4)
```

References

Based on the algorithm presented in [GD]

dwave_networkx.algorithms.elimination_ordering.minor_min_width

minor_min_width(*G*)

Computes a lower bound for the treewidth of graph G.

Parameters **G** (*NetworkX graph*) – The graph on which to compute a lower bound on the treewidth.

Returns **lb** – A lower bound on the treewidth.

Return type int

Examples

This example computes a lower bound for the treewidth of the K_7 complete graph.

```
>>> K_7 = nx.complete_graph(7)
>>> dnx.minor_min_width(K_7)
6
```

References

Based on the algorithm presented in [GD]

dwave_networkx.algorithms.elimination_ordering.min_fill_heuristic

min_fill_heuristic(*G*)

Computes an upper bound on the treewidth of graph G based on the min-fill heuristic for the elimination ordering.

Parameters **G** (*NetworkX graph*) – The graph on which to compute an upper bound for the treewidth.

Returns

- **treewidth_upper_bound** (*int*) – An upper bound on the treewidth of the graph G.
- **order** (*list*) – An elimination order that induces the treewidth.

Examples

This example computes an upper bound for the treewidth of the K_4 complete graph.

```
>>> K_4 = nx.complete_graph(4)
>>> tw, order = dnx.min_fill_heuristic(K_4)
```

References

Based on the algorithm presented in [GD]

dwave_networkx.algorithms.elimination_ordering.min_width_heuristic

min_width_heuristic(*G*)

Computes an upper bound on the treewidth of graph G based on the min-width heuristic for the elimination ordering.

Parameters **G** (*NetworkX graph*) – The graph on which to compute an upper bound for the treewidth.

Returns

- **treewidth_upper_bound** (*int*) – An upper bound on the treewidth of the graph G.
- **order** (*list*) – An elimination order that induces the treewidth.

Examples

This example computes an upper bound for the treewidth of the K_4 complete graph.

```
>>> K_4 = nx.complete_graph(4)
>>> tw, order = dnx.min_width_heuristic(K_4)
```

References

Based on the algorithm presented in [GD]

dwave_networkx.algorithms.elimination_ordering.pegasus_elimination_order

pegasus_elimination_order(*n*, *coordinates=False*)
Provides a variable elimination order for the Pegasus graph.

The treewidth of a Pegasus graph `pegasus_graph(n)` is lower-bounded by $12n - 11$ and upper bounded by $12n - 4$ [BBRR] .

Simple pegasus variable elimination order rules:

- eliminate vertical qubits, one column at a time
- eliminate horizontal qubits in each column once their adjacent vertical qubits have been eliminated

Parameters

- **n** (*int*) – The size parameter for the Pegasus lattice.
- **coordinates** (*bool, optional (default False)*) – If True, the elimination order is given in terms of 4-term Pegasus coordinates, otherwise given in linear indices.

Returns **order** – An elimination order that provides an upper bound on the treewidth.

Return type list

dwave_networkx.algorithms.elimination_ordering.treewidth_branch_and_bound

treewidth_branch_and_bound(*G*, *elimination_order=None*, *treewidth_upperbound=None*)
Computes the treewidth of graph G and a corresponding perfect elimination ordering.

Algorithm based on [GD].

Parameters

- **G** (*NetworkX graph*) – The graph on which to compute the treewidth and perfect elimination ordering.
- **elimination_order** (*list (optional, Default None)*) – An elimination order used as an initial best-known order. If a good order is provided, it may speed up computation. If not provided, the initial order is generated using the min-fill heuristic.
- **treewidth_upperbound** (*int (optional, Default None)*) – An upper bound on the treewidth. Note that using this parameter can result in no returned order.

Returns

- **treewidth** (*int*) – The treewidth of graph G.
- **order** (*list*) – An elimination order that induces the treewidth.

Examples

This example computes the treewidth for the K_7 complete graph using an optionally provided elimination order (a sequential ordering of the nodes, arbitrally chosen).

```
>>> K_7 = nx.complete_graph(7)
>>> dnx.treewidth_branch_and_bound(K_7, [0, 1, 2, 3, 4, 5, 6])
(6, [0, 1, 2, 3, 4, 5, 6])
```

References

Based on the algorithm presented in [GD]

References

Markov Networks

sample_markov_network(MN[, sampler, ...])	Samples from a markov network using the provided sampler.
markov_network_bqm(MN)	Construct a binary quadratic model for a markov network.

dwave_networkx.algorithms.markov.sample_markov_network

sample_markov_network(*MN*, *sampler=None*, *fixed_variables=None*, *return_sampleset=False*, ***sampler_args*)

Samples from a markov network using the provided sampler.

Parameters

- **G** (*NetworkX graph*) – A Markov Network as returned by *markov_network()*
- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **fixed_variables** (*dict*) – A dictionary of variable assignments to be fixed in the markov network.
- **return_sampleset** (*bool (optional, default=False)*) – If True, returns a dimod.SampleSet rather than a list of samples.
- ****sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **samples** – A list of samples ordered from low-to-high energy or a sample set.

Return type list[dict]/dimod.SampleSet

Examples

```
>>> import dimod
...
>>> potentials = {('a', 'b'): {(0, 0): -1,
...                            (0, 1): .5,
```

(continues on next page)

(continued from previous page)

```
...                             (1, 0): .5,
...                             (1, 1): 2}}
>>> MN = dnx.markov_network(potentials)
>>> sampler = dimod.ExactSolver()
>>> samples = dnx.sample_markov_network(MN, sampler)
>>> samples[0]      # doctest: +SKIP
{'a': 0, 'b': 0}
```

```
>>> import dimod
...
>>> potentials = {('a', 'b'): {(0, 0): -1,
...                            (0, 1): .5,
...                            (1, 0): .5,
...                            (1, 1): 2}}
>>> MN = dnx.markov_network(potentials)
>>> sampler = dimod.ExactSolver()
>>> samples = dnx.sample_markov_network(MN, sampler, return_sampleset=True)
>>> samples.first        # doctest: +SKIP
Sample(sample={'a': 0, 'b': 0}, energy=-1.0, num_occurrences=1)
```

```
>>> import dimod
...
>>> potentials = {('a', 'b'): {(0, 0): -1,
...                            (0, 1): .5,
...                            (1, 0): .5,
...                            (1, 1): 2},
...                ('b', 'c'): {(0, 0): -9,
...                            (0, 1): 1.2,
...                            (1, 0): 7.2,
...                            (1, 1): 5}}
>>> MN = dnx.markov_network(potentials)
>>> sampler = dimod.ExactSolver()
>>> samples = dnx.sample_markov_network(MN, sampler, fixed_variables={'b': 0})
>>> samples[0]           # doctest: +SKIP
{'a': 0, 'c': 0, 'b': 0}
```

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

dwave_networkx.algorithms.markov.markov_network_bqm

markov_network_bqm(*MN*)

Construct a binary quadratic model for a markov network.

Parameters **G** (*NetworkX graph*) – A Markov Network as returned by *markov_network()*

Returns **bqm** – A binary quadratic model.

Return type dimod.BinaryQuadraticModel

Matching

A matching is a subset of graph edges in which no vertex occurs more than once.

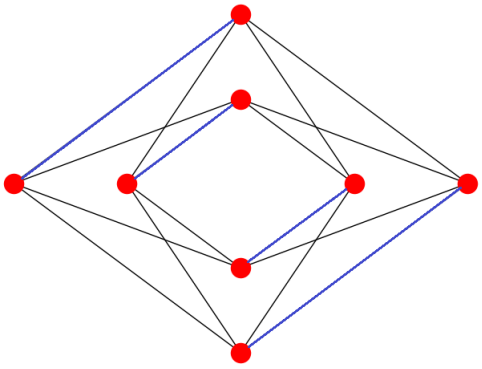


Fig. 3: A matching for a Chimera unit cell: no vertex is incident to more than one edge in the set of blue edges

matching_bqm(G)	Find a binary quadratic model for the graph's matchings.
maximal_matching_bqm(G[, lagrange])	Find a binary quadratic model for the graph's maximal matchings.
min_maximal_matching_bqm(G[, ...])	Find a binary quadratic model for the graph's minimum maximal matchings.
min_maximal_matching(G[, sampler])	Returns an approximate minimum maximal matching.

dwave_networkx.algorithms.matching.matching_bqm

matching_bqm(*G*)

Find a binary quadratic model for the graph's matchings.

A matching is a subset of edges in which no node occurs more than once. This function returns a binary quadratic model (BQM) with ground states corresponding to the possible matchings of G.

Finding valid matchings can be done in polynomial time, so finding matching with BQMs is generally inefficient. This BQM may be useful when combined with other constraints and objectives.

Parameters **G** (*NetworkX graph*) – The graph on which to find a matching.

Returns **bqm** – A binary quadratic model with ground states corresponding to a matching. The variables of the BQM are the edges of *G* as frozensets. The BQM's ground state energy is 0 by construction. The energy of the first excited state is 1.

Return type dimod.BinaryQuadraticModel

dwave_networkx.algorithms.matching.maximal_matching_bqm

maximal_matching_bqm(*G*, *lagrange=None*)

Find a binary quadratic model for the graph's maximal matchings.

A matching is a subset of edges in which no node occurs more than once. A maximal matching is one in which no edges from G can be added without violating the matching rule. This function returns a binary quadratic

model (BQM) with ground states corresponding to the possible maximal matchings of G.

Finding maximal matchings can be done in polynomial time, so finding maximal matching with BQMs is generally inefficient. This BQM may be useful when combined with other constraints and objectives.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a maximal matching.
- **lagrange** (*float (optional)*) – The Lagrange multiplier for the matching constraint. Should be positive and greater than *max_degree - 2*. Defaults to *1.25 * (max_degree - 2)*.

Returns **bqm** – A binary quadratic model with ground states corresponding to a maximal matching. The variables of the BQM are the edges of *G* as frozensets. The BQM's ground state energy is 0 by construction.

Return type dimod.BinaryQuadraticModel

dwave_networkx.algorithms.matching.min_maximal_matching_bqm

min_maximal_matching_bqm(*G*, *maximal_lagrange=2*, *matching_lagrange=None*)

Find a binary quadratic model for the graph's minimum maximal matchings.

A matching is a subset of edges in which no node occurs more than once. A maximal matching is one in which no edges from G can be added without violating the matching rule. A minimum maximal matching is a maximal matching that contains the smallest possible number of edges. This function returns a binary quadratic model (BQM) with ground states corresponding to the possible maximal matchings of G.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a minimum maximal matching.
- **maximal_lagrange** (*float (optional, default=2)*) – The Lagrange multiplier for the maximal constraint. Should be greater than 1.
- **matching_lagrange** (*float (optional)*) – The Lagrange multiplier for the matching constraint. Should be positive and greater than *maximal_lagrange * max_degree - 2*. Defaults to *1.25 * (maximal_lagrange * max_degree - 2)*.

Returns **bqm** – A binary quadratic model with ground states corresponding to a minimum maximal matching. The variables of the BQM are the edges of *G* as frozensets.

Return type dimod.BinaryQuadraticModel

dwave_networkx.algorithms.matching.min_maximal_matching

min_maximal_matching(*G*, *sampler=None*, ***sampler_args*)

Returns an approximate minimum maximal matching.

Defines a QUBO with ground states corresponding to a minimum maximal matching and uses the sampler to sample from it.

A matching is a subset of edges in which no node occurs more than once. A maximal matching is one in which no edges from G can be added without violating the matching rule. A minimum maximal matching is the smallest maximal matching for G.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a minimum maximal matching.

- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **matching** – A minimum maximal matching of the graph.

Return type set

Example

This example uses a sampler from dimod to find a minimum maximal matching for a Chimera unit cell.

```
>>> import dimod
>>> sampler = dimod.ExactSolver()
>>> G = dnx.chimera_graph(1, 1, 4)
>>> matching = dnx.min_maximal_matching(G, sampler)
```

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

References

Matching on Wikipedia

QUBO on Wikipedia

Maximum Cut

A maximum cut is a subset of a graph's vertices such that the number of edges between this subset and the remaining vertices is as large as possible.

maximum_cut(G[, sampler])	Returns an approximate maximum cut.
weighted_maximum_cut(G[, sampler])	Returns an approximate weighted maximum cut.

dwave_networkx.algorithms.max_cut.maximum_cut

maximum_cut(*G*, *sampler=None*, ***sampler_args*)

Returns an approximate maximum cut.

Defines an Ising problem with ground states corresponding to a maximum cut and uses the sampler to sample from it.

A maximum cut is a subset S of the vertices of G such that the number of edges between S and the complementary subset is as large as possible.

Parameters

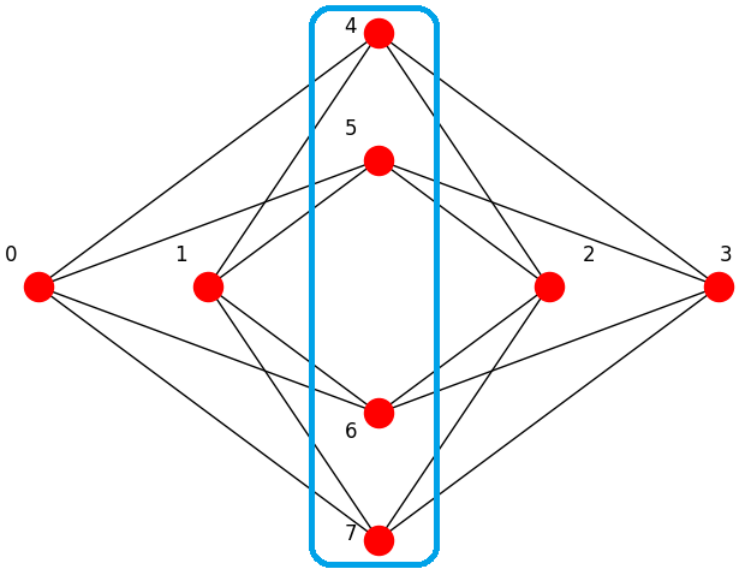


Fig. 4: Maximum cut for a Chimera unit cell: the blue line around the subset of nodes {4, 5, 6, 7} cuts 16 edges; adding or removing a node decreases the number of edges between the two complementary subsets of the graph.

- **G** (*NetworkX graph*) – The graph on which to find a maximum cut.
- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **S** – A maximum cut of G.

Return type set

Example

This example uses a sampler from dimod to find a maximum cut for a graph of a Chimera unit cell created using the *chimera_graph()* function.

```
>>> import dimod
...
>>> sampler = dimod.SimulatedAnnealingSampler()
>>> G = dnx.chimera_graph(1, 1, 4)
>>> cut = dnx.maximum_cut(G, sampler)
```

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

dwave_networkx.algorithms.max_cut.weighted_maximum_cut

weighted_maximum_cut(*G*, *sampler=None*, ***sampler_args*)

Returns an approximate weighted maximum cut.

Defines an Ising problem with ground states corresponding to a weighted maximum cut and uses the sampler to sample from it.

A weighted maximum cut is a subset S of the vertices of G that maximizes the sum of the edge weights between S and its complementary subset.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a weighted maximum cut. Each edge in G should have a numeric *weight* attribute.
- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **S** – A maximum cut of G.

Return type set

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

Independent Set

An independent set is a set of a graph's vertices with no edge connecting any of its member pairs.

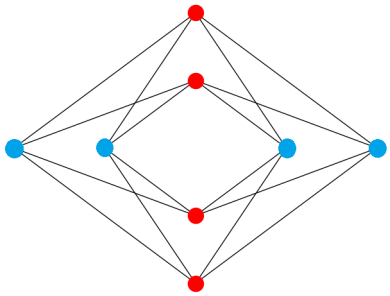


Fig. 5: Independent sets for a Chimera unit cell: the nodes of both the blue set of vertices (the horizontal tile of the Chimera unit cell) and the red set (vertical tile) are independent sets of the graph, with no blue node adjacent to another blue node and likewise for red nodes.

maximum_weighted_independent_set(G[, ...])	Returns an approximate maximum weighted independent set.
maximum_independent_set(G[, sampler, lagrange])	Returns an approximate maximum independent set.
is_independent_set(G, indep_nodes)	Determines whether the given nodes form an independent set.

dwave_networkx.maximum_weighted_independent_set

maximum_weighted_independent_set(*G*, *weight=None*, *sampler=None*, *lagrange=2.0*, ***sampler_args*)

Returns an approximate maximum weighted independent set.

Defines a QUBO with ground states corresponding to a maximum weighted independent set and uses the sampler to sample from it.

An independent set is a set of nodes such that the subgraph of G induced by these nodes contains no edges. A maximum independent set is an independent set of maximum total node weight.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a maximum cut weighted independent set.
- **weight** (*string, optional (default None)*) – If None, every node has equal weight. If a string, use this node attribute as the node weight. A node without this attribute is assumed to have max weight.
- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **lagrange** (*optional (default 2)*) – Lagrange parameter to weight constraints (no edges within set) versus objective (largest set possible).
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **indep_nodes** – List of nodes that form a maximum weighted independent set, as determined by the given sampler.

Return type list

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

References

Independent Set on Wikipedia

QUBO on Wikipedia

dwave_networkx.maximum_independent_set

maximum_independent_set(*G*, *sampler=None*, *lagrange=2.0*, ***sampler_args*)

Returns an approximate maximum independent set.

Defines a QUBO with ground states corresponding to a maximum independent set and uses the sampler to sample from it.

An independent set is a set of nodes such that the subgraph of G induced by these nodes contains no edges. A maximum independent set is an independent set of largest possible size.

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a maximum cut independent set.
- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **lagrange** (*optional (default 2)*) – Lagrange parameter to weight constraints (no edges within set) versus objective (largest set possible).
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **indep_nodes** – List of nodes that form a maximum independent set, as determined by the given sampler.

Return type list

Example

This example uses a sampler from dimod to find a maximum independent set for a graph of a Chimera unit cell created using the *chimera_graph()* function.

```
>>> import dimod
>>> sampler = dimod.SimulatedAnnealingSampler()
>>> G = dnx.chimera_graph(1, 1, 4)
>>> indep_nodes = dnx.maximum_independent_set(G, sampler)
```

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

References

Independent Set on Wikipedia

QUBO on Wikipedia

dwave_networkx.is_independent_set

is_independent_set(*G*, *indep_nodes*)

Determines whether the given nodes form an independent set.

An independent set is a set of nodes such that the subgraph of G induced by these nodes contains no edges.

Parameters

- **G** (*NetworkX graph*) – The graph on which to check the independent set.
- **indep_nodes** (*list*) – List of nodes that form a maximum independent set, as determined by the given sampler.

Returns **is_independent** – True if indep_nodes form an independent set.

Return type bool

Example

This example checks two sets of nodes, both derived from a single Chimera unit cell, for an independent set. The first set is the horizontal tile's nodes; the second has nodes from the horizontal and verical tiles.

```
>>> import dwave_networkx as dnx
>>> G = dnx.chimera_graph(1, 1, 4)
>>> dnx.is_independent_set(G, [0, 1, 2, 3])
True
>>> dnx.is_independent_set(G, [0, 4])
False
```

Helper Functions

maximum_weighted_independent_set_qubo(G[, ...])	Return the QUBO with ground states corresponding to a maximum weighted independent set.

dwave_networkx.algorithms.independent_set.maximum_weighted_independent_set_qubo

maximum_weighted_independent_set_qubo(*G*, *weight=None*, *lagrange=2.0*)

Return the QUBO with ground states corresponding to a maximum weighted independent set.

Parameters

- **G** (*NetworkX graph*) –
- **weight** (*string, optional (default None)*) – If None, every node has equal weight. If a string, use this node attribute as the node weight. A node without this attribute is assumed to have max weight.
- **lagrange** (*optional (default 2)*) – Lagrange parameter to weight constraints (no edges within set) versus objective (largest set possible).

Returns **QUBO** – The QUBO with ground states corresponding to a maximum weighted independent set.

Return type dict

Examples

```
>>> from dwave_networkx.algorithms.independent_set import maximum_weighted_
↪independent_set_qubo
...
>>> G = nx.path_graph(3)
>>> Q = maximum_weighted_independent_set_qubo(G, weight='weight', lagrange=2.0)
>>> Q[(0, 0)]
-1.0
>>> Q[(1, 1)]
-1.0
>>> Q[(0, 1)]
2.0
```

Partitioning

A *k*-partition consists of *k* disjoint and equally sized subsets of a graph's vertices such that the total number of edges between nodes in distinct subsets is as small as possible.

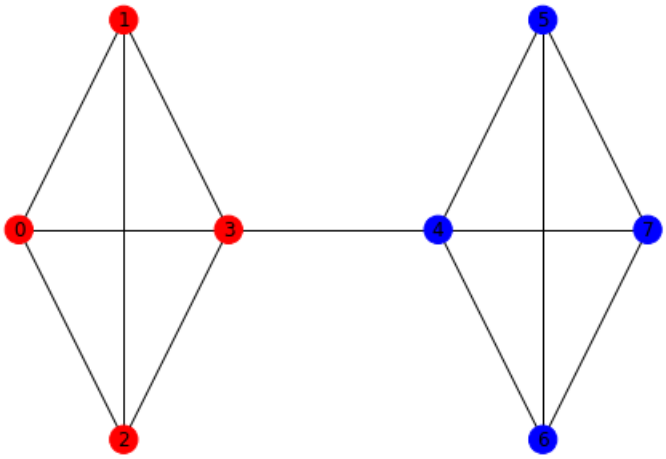


Fig. 6: A 2-partition for a simple graph: the nodes in blue are in the '0' subset, and the nodes in red are in the '1' subset. There are no other arrangements with fewer edges between two equally sized subsets.

partition(G[, num_partitions, sampler])	Returns an approximate k-partition of G.

dwave_networkx.algorithms.partition.partition

partition(*G*, *num_partitions=2*, *sampler=None*, ***sampler_args*)

Returns an approximate k-partition of G.

Defines an CQM with ground states corresponding to a balanced k-partition of G and uses the sampler to sample from it. A k-partition is a collection of k subsets of the vertices of G such that each vertex is in exactly one subset, and the number of edges between vertices in different subsets is as small as possible. If G is a weighted graph, the sum of weights over those edges are minimized.

Parameters

- **G** (*NetworkX graph*) – The graph to partition.
- **num_partitions** (*int, optional (default 2)*) – The number of subsets in the desired partition.
- **sampler** – A constrained quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Model, with or without constraints. The sampler is expected to have a 'sample_cqm' method. A sampler is expected to return an iterable of samples, in order of increasing energy.
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **node_partition** – The partition as a dictionary mapping each node to subsets labelled as integers 0, 1, 2, ... num_partitions.

Return type dict

Example

This example uses a sampler from dimod to find a 2-partition for a graph of a Chimera unit cell created using the *chimera_graph()* function.

```
>>> import dimod
>>> sampler = dimod.ExactCQMSolver()
>>> G = dnx.chimera_graph(1, 1, 4)
>>> partitions = dnx.partition(G, sampler=sampler)
```

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

Social

A signed social network graph is a graph whose signed edges represent friendly/hostile interactions between vertices.

structural_imbalance(S[, sampler])	Returns an approximate set of frustrated edges and a bicoloring.
structural_imbalance_ising(S)	Construct the Ising problem to calculate the structural imbalance of a signed social network.

dwave_networkx.algorithms.social.structural_imbalance

structural_imbalance(*S*, *sampler=None*, ***sampler_args*)

Returns an approximate set of frustrated edges and a bicoloring.

A signed social network graph is a graph whose signed edges represent friendly/hostile interactions between nodes. A signed social network is considered balanced if it can be cleanly divided into two factions, where all relations within a faction are friendly, and all relations between factions are hostile. The measure of imbalance or frustration is the minimum number of edges that violate this rule.

Parameters

- **S** (*NetworkX graph*) – A social graph on which each edge has a 'sign' attribute with a

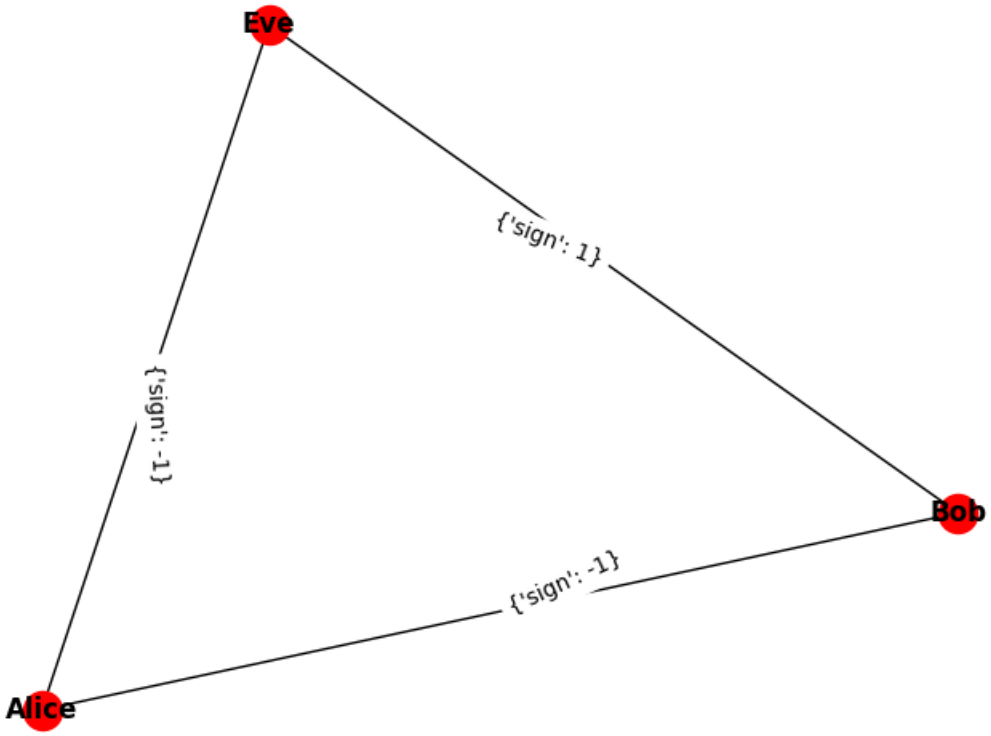


Fig. 7: A signed social graph for three nodes, where Eve and Bob are friendly with each other and hostile to Alice. This network is balanced because it can be cleanly divided into two subsets, {Bob, Eve} and {Alice}, with friendly relations within each subset and only hostile relations between the subsets.

numeric value.

- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrainted Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns

- **frustrated_edges** (*dict*) – A dictionary of the edges that violate the edge sign. The imbalance of the network is the length of frustrated_edges.
- **colors** (*dict*) – A bicoloring of the nodes into two factions.

Raises `ValueError` – If any edge does not have a 'sign' attribute.

Examples

```
>>> import dimod
>>> sampler = dimod.ExactSolver()
>>> S = nx.Graph()
>>> S.add_edge('Alice', 'Bob', sign=1)  # Alice and Bob are friendly
>>> S.add_edge('Alice', 'Eve', sign=-1)  # Alice and Eve are hostile
>>> S.add_edge('Bob', 'Eve', sign=-1)  # Bob and Eve are hostile
>>> frustrated_edges, colors = dnx.structural_imbalance(S, sampler)
>>> print(frustrated_edges)
{}
>>> print(colors)  # doctest: +SKIP
{'Alice': 0, 'Bob': 0, 'Eve': 1}
>>> S.add_edge('Ted', 'Bob', sign=1)  # Ted is friendly with all
>>> S.add_edge('Ted', 'Alice', sign=1)
>>> S.add_edge('Ted', 'Eve', sign=1)
>>> frustrated_edges, colors = dnx.structural_imbalance(S, sampler)
>>> print(frustrated_edges)  # doctest: +SKIP
{('Ted', 'Eve'): {'sign': 1}}
>>> print(colors)  # doctest: +SKIP
{'Bob': 1, 'Ted': 1, 'Alice': 1, 'Eve': 0}
```

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

References

Ising model on Wikipedia

dwave_networkx.algorithms.social.structural_imbalance_ising

structural_imbalance_ising(*S*)

Construct the Ising problem to calculate the structural imbalance of a signed social network.

A signed social network graph is a graph whose signed edges represent friendly/hostile interactions between nodes. A signed social network is considered balanced if it can be cleanly divided into two factions, where all relations within a faction are friendly, and all relations between factions are hostile. The measure of imbalance or frustration is the minimum number of edges that violate this rule.

Parameters **S** (*NetworkX graph*) – A social graph on which each edge has a 'sign' attribute with a numeric value.

Returns

- **h** (*dict*) – The linear biases of the Ising problem. Each variable in the Ising problem represent a node in the signed social network. The solution that minimized the Ising problem will assign each variable a value, either -1 or 1. This bi-coloring defines the factions.
- **J** (*dict*) – The quadratic biases of the Ising problem.

Raises ValueError – If any edge does not have a 'sign' attribute.

Examples

```
>>> import dimod
>>> from dwave_networkx.algorithms.social import structural_imbalance_ising
...
>>> S = nx.Graph()
>>> S.add_edge('Alice', 'Bob', sign=1)  # Alice and Bob are friendly
>>> S.add_edge('Alice', 'Eve', sign=-1)  # Alice and Eve are hostile
>>> S.add_edge('Bob', 'Eve', sign=-1)  # Bob and Eve are hostile
...
>>> h, J = structural_imbalance_ising(S)
>>> h  # doctest: +SKIP
{'Alice': 0.0, 'Bob': 0.0, 'Eve': 0.0}
>>> J  # doctest: +SKIP
{('Alice', 'Bob'): -1.0, ('Alice', 'Eve'): 1.0, ('Bob', 'Eve'): 1.0}
```

Traveling Salesperson

A traveling salesperson route is an ordering of the vertices in a complete weighted graph.

traveling_salesperson(G[, sampler, ...])	Returns an approximate minimum traveling salesperson route.
traveling_salesperson_qubo(G[, lagrange, ...])	Return the QUBO with ground states corresponding to a minimum TSP route.

dwave_networkx.algorithms.tsp.traveling_salesperson

traveling_salesperson(*G*, *sampler=None*, *lagrange=None*, *weight='weight'*, *start=None*, ***sampler_args*)

Returns an approximate minimum traveling salesperson route.

Defines a QUBO with ground states corresponding to the minimum routes and uses the sampler to sample from it.

A route is a cycle in the graph that reaches each node exactly once. A minimum route is a route with the smallest total edge weight.

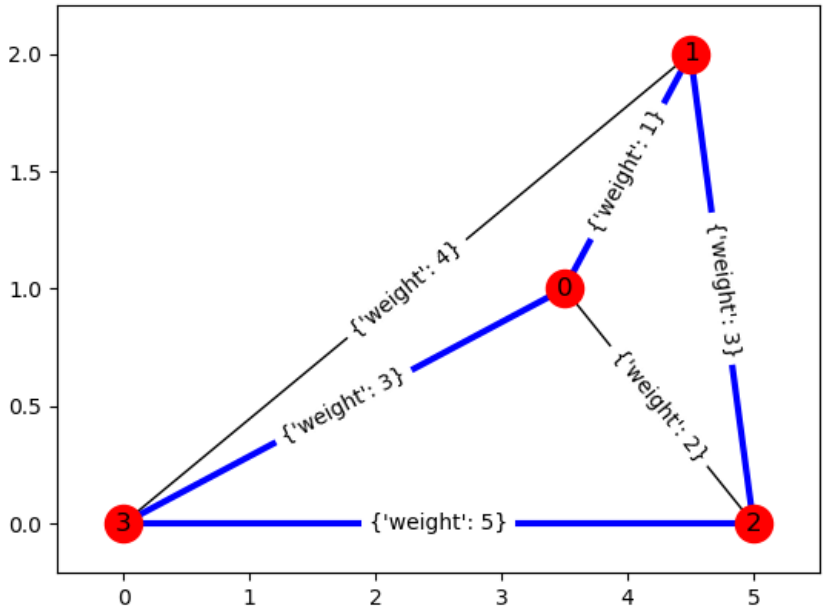


Fig. 8: A traveling salesperson route of [2, 1, 0, 3].

Parameters

- **G** (*NetworkX graph*) – The graph on which to find a minimum traveling salesperson route. This should be a complete graph with non-zero weights on every edge.
- **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy. If no sampler is provided, one must be provided using the *set_default_sampler* function.
- **lagrange** (*number, optional (default None)*) – Lagrange parameter to weight constraints (visit every city once) versus objective (shortest distance route).
- **weight** (*optional (default 'weight')*) – The name of the edge attribute containing the weight.
- **start** (*node, optional*) – If provided, the route will begin at *start*.
- **sampler_args** – Additional keyword parameters are passed to the sampler.

Returns **route** – List of nodes in order to be visited on a route

Return type list

Examples

```
>>> import dimod
...
>>> G = nx.Graph()
>>> G.add_weighted_edges_from({(0, 1, .1), (0, 2, .5), (0, 3, .1), (1, 2, .1),
...                            (1, 3, .5), (2, 3, .1)})
>>> dnx.traveling_salesperson(G, dimod.ExactSolver(), start=0) # doctest: +SKIP
[0, 1, 2, 3]
```

Notes

Samplers by their nature may not return the optimal solution. This function does not attempt to confirm the quality of the returned sample.

dwave_networkx.algorithms.tsp.traveling_salesperson_qubo

traveling_salesperson_qubo(*G*, *lagrange=None*, *weight='weight'*, *missing_edge_weight=None*)

Return the QUBO with ground states corresponding to a minimum TSP route.

If $|G|$ is the number of nodes in the graph, the resulting qubo will have:

- $|G|^2$ variables/nodes
- $2|G|^2(|G|-1)$ interactions/edges

Parameters

- **G** (*NetworkX graph*) – A complete graph in which each edge has a attribute giving its weight.
- **lagrange** (*number, optional (default None)*) – Lagrange parameter to weight constraints (no edges within set) versus objective (largest set possible).
- **weight** (*optional (default 'weight')*) – The name of the edge attribute containing the weight.
- **missing_edge_weight** (*number, optional (default None)*) – For bidirectional graphs, the weight given to missing edges. If None is given (the default), missing edges will be set to the sum of all weights.

Returns **QUBO** – The QUBO with ground states corresponding to a minimum travelling salesperson route. The QUBO variables are labelled *(c, t)* where *c* is a node in *G* and *t* is the time index. For instance, if *('a', 0)* is 1 in the ground state, that means the node 'a' is visted first.

Return type dict

Drawing

Tools to visualize topologies of D-Wave QPUs and weighted graph problems on them.

Note: Some functionality requires NumPy and/or Matplotlib.

Chimera Graph Functions

Tools to visualize Chimera lattices and weighted graph problems on them.

chimera_layout(G[, scale, center, dim])	Positions the nodes of graph G in a Chimera cross topology.
draw_chimera(G, **kwargs)	Draws graph G in a Chimera cross topology.

dwave_networkx.drawing.chimera_layout.chimera_layout

chimera_layout(*G*, *scale=1.0*, *center=None*, *dim=2*)

Positions the nodes of graph G in a Chimera cross topology.

NumPy (https://scipy.org) is required for this function.

Parameters

- **G** (*NetworkX graph*) – Should be a Chimera graph or a subgraph of a Chimera graph. If every node in G has a *chimera_index* attribute, those are used to place the nodes. Otherwise makes a best-effort attempt to find positions.
- **scale** (*float (default 1.)*) – Scale factor. When scale = 1, all positions fit within [0, 1] on the x-axis and [-1, 0] on the y-axis.
- **center** (*None or array (default None)*) – Coordinates of the top left corner.
- **dim** (*int (default 2)*) – Number of dimensions. When dim > 2, all extra dimensions are set to 0.

Returns **pos** – A dictionary of positions keyed by node.

Return type dict

Examples

```
>>> G = dnx.chimera_graph(1)
>>> pos = dnx.chimera_layout(G)
```

dwave_networkx.drawing.chimera_layout.draw_chimera

draw_chimera(*G*, ***kwargs*)

Draws graph G in a Chimera cross topology.

If *linear_biases* and/or *quadratic_biases* are provided, these are visualized on the plot.

Parameters

- **G** (*NetworkX graph*) – Should be a Chimera graph or a subgraph of a Chimera graph.
- **linear_biases** (*dict (optional, default {})*) – A dict of biases associated with each node in G. Should be of form {node: bias, ... }. Each bias should be numeric.
- **quadratic_biases** (*dict (optional, default {})*) – A dict of biases associated with each edge in G. Should be of form {edge: bias, ... }. Each bias should be numeric. Self-loop edges (i.e., $i = j$) are treated as linear biases.

- **kwargs** (*optional keywords*) – See networkx.draw_networkx() for a description of optional keywords, with the exception of the *pos* parameter which is not used by this function. If *linear_biases* or *quadratic_biases* are provided, any provided *node_color* or *edge_color* arguments are ignored.

Examples

```
>>> # Plot 2x2 Chimera unit cells
>>> import networkx as nx
>>> import dwave_networkx as dnx
>>> import matplotlib.pyplot as plt  # doctest: +SKIP
>>> G = dnx.chimera_graph(2, 2, 4)
>>> dnx.draw_chimera(G)  # doctest: +SKIP
>>> plt.show()  # doctest: +SKIP
```

Example

This example uses the *chimera_layout()* function to show the positions of nodes of a simple 5-node NetworkX graph in a Chimera lattice. It then uses the *chimera_graph()* and *draw_chimera()* functions to display those positions on a Chimera unit cell.

```
>>> import networkx as nx
>>> import dwave_networkx as dnx
>>> import matplotlib.pyplot as plt
>>> H = nx.Graph()
>>> H.add_nodes_from([0, 4, 5, 6, 7])
>>> H.add_edges_from([(0, 4), (0, 5), (0, 6), (0, 7)])
>>> pos=dnx.chimera_layout(H)
>>> pos
{0: array([ 0. , -0.5]),
 4: array([ 0.5,  0. ]),
 5: array([ 0.5 , -0.25]),
 6: array([ 0.5 , -0.75]),
 7: array([ 0.5, -1. ])}
>>> # Show graph H on a Chimera unit cell
>>> plt.ion()
>>> G=dnx.chimera_graph(1, 1, 4)  # Draw a Chimera unit cell
>>> dnx.draw_chimera(G)
>>> dnx.draw_chimera(H, node_color='b', node_shape='*', style='dashed', edge_color='b
↪', width=3)
>>> # matplotlib commands to add labels to graphic not shown
```

Pegasus Graph Functions

Tools to visualize Pegasus lattices and weighted graph problems on them.

draw_pegasus(G[, crosses])	Draws graph G in a Pegasus topology.
draw_pegasus_embedding(G, *args, **kwargs)	Draws an embedding onto Pegasus graph G.
pegasus_layout(G[, scale, center, dim, crosses])	Positions the nodes of graph G in a Pegasus topology.
pegasus_node_placer_2d(G[, scale, center, ...])	Generates a function to convert Pegasus indices to plottable coordinates.

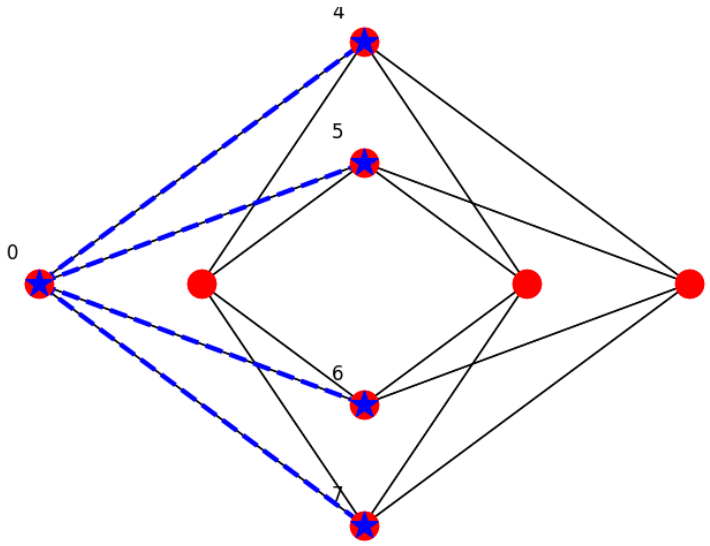


Fig. 9: Graph H (blue) overlaid on a Chimera unit cell (red nodes and black edges).

dwave_networkx.drawing.pegasus_layout.draw_pegasus

draw_pegasus(*G*, *crosses=False*, ***kwargs*)

Draws graph G in a Pegasus topology.

If `linear_biases` and/or `quadratic_biases` are provided, these are visualized on the plot.

Parameters

- **G** (*NetworkX graph*) – A Pegasus graph or a subgraph of a Pegasus graph, as produced by the *dwave_networkx.pegasus_graph()* function.
- **linear_biases** (*dict (optional, default {})*) – Biases as a dict, of form {node: bias, ... }, where keys are nodes in G and biases are numeric.
- **quadratic_biases** (*dict (optional, default {})*) – Biases as a dict, of form {edge: bias, ... }, where keys are edges in G and biases are numeric. Self-loop edges (i.e., $i = j$) are treated as linear biases.
- **crosses** (*boolean (optional, default False)*) – If True, $K_{4,4}$ subgraphs are shown in a cross rather than L configuration. Ignored if G is defined with `nice_coordinates=True`.
- **kwargs** (*optional keywords*) – See networkx.draw_networkx() for a description of optional keywords, with the exception of the `pos` parameter, which is not used by this function. If `linear_biases` or `quadratic_biases` are provided, any provided `node_color` or `edge_color` arguments are ignored.

Examples

This example plots a Pegasus graph with size parameter 2.

```
>>> import networkx as nx
>>> import dwave_networkx as dnx
>>> import matplotlib.pyplot as plt    # doctest: +SKIP
```

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```
>>> G = dnx.pegasus_graph(2)
>>> dnx.draw_pegasus(G)     # doctest: +SKIP
>>> plt.show()     # doctest: +SKIP
```

dwave_networkx.drawing.pegasus_layout.draw_pegasus_embedding

draw_pegasus_embedding(*G*, **args*, ***kwargs*)

Draws an embedding onto Pegasus graph G.

Parameters

- **G** (*NetworkX graph*) – A Pegasus graph or a subgraph of a Pegasus graph, as produced by the *dwave_networkx.pegasus_graph()* function.
- **emb** (*dict*) – Chains, as a dict of form {qubit: chain, ...}, where qubits are nodes in G and chains are iterables of qubit labels.
- **embedded_graph** (*NetworkX graph (optional, default None)*) – A graph that contains all keys of `emb` as nodes. If specified, edges of G are considered interactions if and only if (1) they exist between two chains of `emb` and (2) their keys are connected by an edge in this graph. If given, only couplers between chains based on this graph are displayed.
- **interaction_edges** (*list (optional, default None)*) – A list of edges used as interactions. If given, only these couplers are displayed.
- **show_labels** (*boolean (optional, default False)*) – If True, each chain in `emb` is labelled with its key.
- **chain_color** (*dict (optional, default None)*) – Colors as a dict of form {node: rgba_color, ...} associated with each key in `emb`, where colors are length-4 tuples of floats between 0 and 1 inclusive. If None, each chain is assigned a different color.
- **unused_color** (*tuple (optional, default (0.9,0.9,0.9,1.0))*) – Color for nodes of G that are not part of chains, and edges that are neither chain edges nor interactions. If None, these nodes and edges are not shown.
- **crosses** (*boolean (optional, default False)*) – If True, $K_{4,4}$ subgraphs are shown in a cross rather than L configuration. Ignored if G is defined with `nice_coordinates=True`.
- **overlapped_embedding** (*boolean (optional, default False)*) – If True, chains in `emb` may overlap (contain the same vertices in G), and these overlaps are displayed as concentric circles.
- **kwargs** (*optional keywords*) – See networkx.draw_networkx() for a description of optional keywords, with the exception of the `pos` parameter, which is not used by this function. If `linear_biases` or `quadratic_biases` are provided, any provided `node_color` or `edge_color` arguments are ignored.

dwave_networkx.drawing.pegasus_layout.pegasus_layout

pegasus_layout(*G*, *scale=1.0*, *center=None*, *dim=2*, *crosses=False*)

Positions the nodes of graph G in a Pegasus topology.

NumPy is required for this function.

Parameters

- **G** (*NetworkX graph*) – A Pegasus graph or a subgraph of a Pegasus graph, as produced by the *dwave_networkx.pegasus_graph()* function.
- **scale** (*float (default 1.)*) – Scale factor. A setting of `scale = 1` fits all positions within [0, 1] on the x-axis and [-1, 0] on the y-axis.
- **center** (*None or array (default None)*) – Coordinates of the top left corner.
- **dim** (*int (default 2)*) – Number of dimensions. When dim > 2, all extra dimensions are set to 0.
- **crosses** (*boolean (optional, default False)*) – If True, $K_{4,4}$ subgraphs are shown in a cross rather than L configuration. Ignored if G is defined with `nice_coordinates=True`.

Returns **pos** – Positions as a dictionary keyed by node.

Return type dict

Examples

This example gives the positions of a Pegasus lattice of size 2.

```
>>> G = dnx.pegasus_graph(2)
>>> pos = dnx.pegasus_layout(G)
```

dwave_networkx.drawing.pegasus_layout.pegasus_node_placer_2d

pegasus_node_placer_2d(*G*, *scale=1.0*, *center=None*, *dim=2*, *crosses=False*)

Generates a function to convert Pegasus indices to plottable coordinates.

Parameters

- **G** (*NetworkX graph*) – A Pegasus graph or a subgraph of a Pegasus graph, as produced by the *dwave_networkx.pegasus_graph()* function.
- **scale** (*float (default 1.)*) – Scale factor. A setting of `scale = 1` fits all positions within [0, 1] on the x-axis and [-1, 0] on the y-axis.
- **center** (*None or array (default None)*) – Coordinates of the top left corner.
- **dim** (*int (default 2)*) – Number of dimensions. When dim > 2, all extra dimensions are set to 0.
- **crosses** (*boolean (optional, default False)*) – If True, $K_{4,4}$ subgraphs are shown in a cross rather than L configuration.

Returns **xy_coords** – A function that maps a Pegasus index (u, w, k, z) in a Pegasus lattice to plottable x,y coordinates.

Return type function

Example

This example uses the *draw_pegasus()* function to show the positions of nodes of a simple 5-node graph on a small Pegasus lattice.

```
>>> import dwave_networkx as dnx
>>> import matplotlib.pyplot as plt
>>> G = dnx.pegasus_graph(2)
>>> H = dnx.pegasus_graph(2, node_list=[4, 40, 41, 42, 43],
                          edge_list=[(4, 40), (4, 41), (4, 42), (4, 43)])
>>> # Show graph H on a small Pegasus lattice
>>> plt.ion()
>>> # Show graph H on a small Pegasus lattice
>>> plt.ion()
>>> dnx.draw_pegasus(G, with_labels=True, crosses=True, node_color="Yellow")
>>> dnx.draw_pegasus(H, crosses=True, node_color='b', style='dashed',
                     edge_color='b', width=3)
```

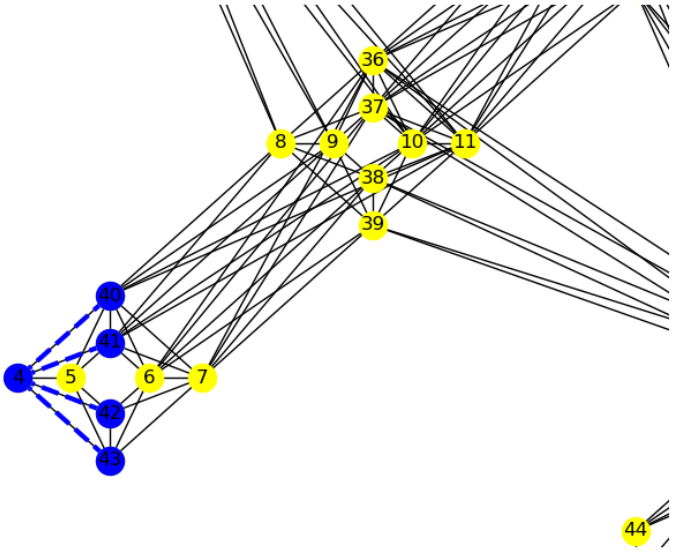

Fig. 10: Graph H (blue) overlaid on a small Pegasus lattice(yellow nodes and black edges).

Zephyr Graph Functions

Tools to visualize Zephyr lattices and weighted graph problems on them.

draw_zephyr(G, **kwargs)	Draws graph G in a Zephyr topology.
draw_zephyr_embedding(G, *args, **kwargs)	Draws an embedding onto Zephyr graph G.
draw_zephyr_yield(G, **kwargs)	Draws the given graph G with highlighted faults, according to layout.
zephyr_layout(G[, scale, center, dim])	Positions the nodes of graph G in a Zephyr topology.

dwave_networkx.drawing.zephyr_layout.draw_zephyr

draw_zephyr(*G*, ***kwargs*)

Draws graph G in a Zephyr topology.

If `linear_biases` and/or `quadratic_biases` are provided, these are visualized on the plot.

Parameters

- **G** (*NetworkX graph*) – A Zephyr graph or a subgraph of a Zephyr graph, as produced by the *dwave_networkx.zephyr_graph()* function.
- **linear_biases** (*dict (optional, default {})*) – Biases as a dict, of form {node: bias, ...}, where keys are nodes in G and biases are numeric.
- **quadratic_biases** (*dict (optional, default {})*) – Biases as a dict, of form {edge: bias, ...}, where keys are edges in G and biases are numeric. Self-loop edges (i.e., $i = j$) are treated as linear biases.
- **kwargs** (*optional keywords*) – See `draw_networkx()` for a description of optional keywords, with the exception of the `pos` parameter, which is unsupported. If the `linear_biases` or `quadratic_biases` parameters are provided, any provided `node_color` or `edge_color` arguments are ignored.

Examples

This example plots a Zephyr graph with size parameter 2.

```
>>> import networkx as nx
>>> import dwave_networkx as dnx
>>> import matplotlib.pyplot as plt    # doctest: +SKIP
>>> G = dnx.zephyr_graph(2)
>>> dnx.draw_zephyr(G)     # doctest: +SKIP
>>> plt.show()     # doctest: +SKIP
```

dwave_networkx.drawing.zephyr_layout.draw_zephyr_embedding

draw_zephyr_embedding(*G*, **args*, ***kwargs*)

Draws an embedding onto Zephyr graph G.

Parameters

- **G** (*NetworkX graph*) – A Zephyr graph or a subgraph of a Zephyr graph, as produced by the *dwave_networkx.zephyr_graph()* function.
- **emb** (*dict*) – Minor-embedding as a dict of form {node: chain, ...}, where `node` are nodes in G and `chain` are iterables of qubit labels.
- **embedded_graph** (*NetworkX graph (optional, default None)*) – A graph that contains all keys of `emb` as nodes. If specified, edges of G are considered interactions if and only if (1) they exist between two chains of `emb` and (2) the keys of the corresponding chains are connected by an edge in the given graph. If given, only couplers between chains based on this graph are displayed.
- **interaction_edges** (*list (optional, default None)*) – A list of edges used as interactions. If given, only these couplers are displayed.
- **show_labels** (*boolean (optional, default False)*) – If True, each chain in `emb` is labelled with its key.
- **chain_color** (*dict (optional, default None)*) – Colors as a dict of form {node: rgba_color, ...} associated with each key in `emb`, where colors are length-4 tuples of floats between 0 and 1 inclusive. If None, each chain is assigned a different color.
- **unused_color** (*tuple (optional, default (0.9,0.9,0.9,1.0))*) – Color for nodes of G that are not part of chains, and edges that are neither chain edges nor interactions. If None, these nodes and edges are not shown.

- **overlapped_embedding** (*boolean (optional, default False)*) – If True, chains in `emb` may overlap (contain the same vertices in G), and these overlaps are displayed as concentric circles.
- **kwargs** (*optional keywords*) – See `draw_networkx()` for a description of optional keywords, with the exception of the `pos` parameter, which is unsupported. If the `linear_biases` or `quadratic_biases` parameters are provided, any provided `node_color` or `edge_color` arguments are ignored.

dwave_networkx.drawing.zephyr_layout.draw_zephyr_yield

draw_zephyr_yield(*G*, ***kwargs*)

Draws the given graph G with highlighted faults, according to layout.

Parameters

- **G** (*NetworkX graph*) – Graph to be parsed for faults.
- **unused_color** (*tuple or color string (optional, default (0.9, 0.9, 0.9, 1.0))*) – The color to use for nodes and edges of G which are not faults. If unused_color is None, these nodes and edges will not be shown at all.
- **fault_color** (*tuple or color string (optional, default (1.0, 0.0, 0.0, 1.0))*) – A color to represent nodes absent from the graph G. Colors should be length-4 tuples of floats between 0 and 1 inclusive.
- **fault_shape** (*string, optional (default='x')*) – The shape of the fault nodes. Specification is as for Matplotlib's markers; for example "o" (circle), "^" (triangle)", "s" (square) and many more options.
- **fault_style** (*string, optional (default='dashed')*) – Edge fault line style (solid|dashed|dotted|dashdot)
- **kwargs** (*optional keywords*) – See `draw_networkx()` for a description of optional keywords, with the exception of the `pos` parameter, which is unsupported. If the `linear_biases` or `quadratic_biases` parameters are provided, any provided `node_color` or `edge_color` arguments are ignored.

dwave_networkx.drawing.zephyr_layout.zephyr_layout

zephyr_layout(*G*, *scale=1.0*, *center=None*, *dim=2*)

Positions the nodes of graph G in a Zephyr topology.

NumPy is required for this function.

Parameters

- **G** (*NetworkX graph*) – A Zephyr graph or a subgraph of a Zephyr graph, as produced by the *dwave_networkx.zephyr_graph()* function.
- **scale** (*float (default 1.)*) – Scale factor. A setting of `scale = 1` fits all positions within [0, 1] on the x-axis and [-1, 0] on the y-axis.
- **center** (*None or array (default None)*) – Coordinates of the top left corner.
- **dim** (*int (default 2)*) – Number of dimensions. When dim > 2, all extra dimensions are set to 0.

Returns **pos** – Positions as a dictionary keyed by node.

Return type dict

Examples

This example gives the positions of a Zephyr lattice of size 2.

```
>>> G = dnx.zephyr_graph(2)
>>> pos = dnx.zephyr_layout(G)
```

Example

This example uses the *draw_zephyr_embedding()* function to show the positions of a five-node clique on a small Zephyr graph.

```
>>> import dwave_networkx as dnx
>>> import matplotlib.pyplot as plt
>>> import networkx as nx
...
>>> G = dnx.zephyr_graph(1)
>>> embedding = {"N1": [13, 44], "N2": [11], "N3": [41], "N4": [40], "N5": [9, 37]}
...
>>> plt.ion()
>>> dnx.draw_zephyr_embedding(G, embedding, show_labels=True)
```

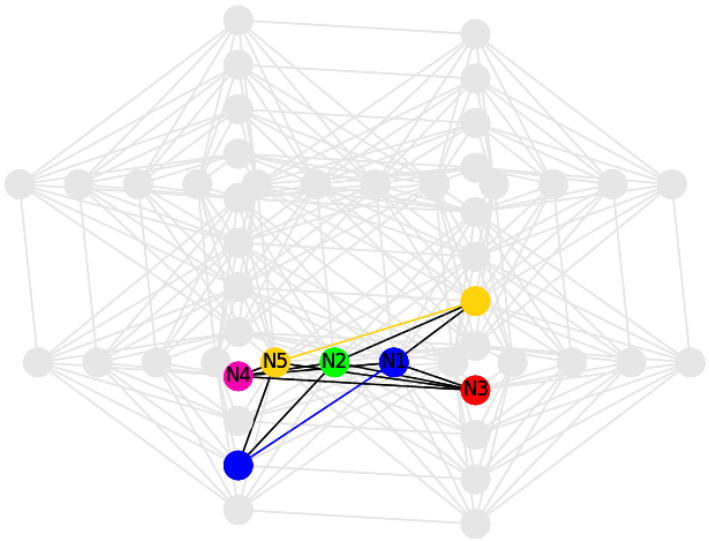

Fig. 11: Five-node clique embedded in a small Zephyr graph.

Graph Generators

Generators for graphs, such the graphs (topologies) of D-Wave System QPUs.

D-Wave Systems

chimera_graph(m[, n, t, create_using, ...])	Creates a Chimera lattice of size (m, n, t).
pegasus_graph(m[, create_using, node_list, ...])	Creates a Pegasus graph with size parameter *m*.
zephyr_graph(m[, t, create_using, ...])	Creates a Zephyr graph with grid parameter `m` and tile parameter `t`.

dwave_networkx.chimera_graph

chimera_graph(*m*, *n=None*, *t=None*, *create_using=None*, *node_list=None*, *edge_list=None*, *data=True*, *coordinates=False*, *check_node_list=False*, *check_edge_list=False*)

Creates a Chimera lattice of size (m, n, t).

Parameters

- **m** (*int*) – Number of rows in the Chimera lattice.
- **n** (*int (optional, default m)*) – Number of columns in the Chimera lattice.
- **t** (*int (optional, default 4)*) – Size of the shore within each Chimera tile.
- **create_using** (*Graph (optional, default None)*) – If provided, this graph is cleared of nodes and edges and filled with the new graph. Usually used to set the type of the graph.
- **node_list** (*iterable (optional, default None)*) – Iterable of nodes in the graph. The nodes should typically be compatible with the requested lattice-shape parameters and coordinate system; incompatible nodes are accepted unless you set `check_node_list=True`. If not specified, calculated from (`m`, `n`, `t`) and `coordinates` per the topology description below; all $2tmn$ nodes are included.
- **edge_list** (*iterable (optional, default None)*) – Iterable of edges in the graph. Edges must be 2-tuples of the nodes specified in `node_list`, or calculated from (`m`, `n`, `t`) and `coordinates` per the topology description below; incompatible edges are ignored unless you set `check_edge_list=True`. If not specified, all edges compatible with the `node_list` and topology description are included.
- **data** (bool (optional, default `True`)) – If `True`, each node has a *chimera_index attribute*. The attribute is a 4-tuple Chimera index as defined below.
- **coordinates** (bool (optional, default `False`)) – If `True`, node labels are 4-tuples, equivalent to the chimera_index attribute as below. In this case, the `data` parameter controls the existence of a *linear_index attribute*, which is an integer.
- **check_node_list** (bool (optional, default `False`)) – If `True`, the `node_list` elements are checked for compatibility with the graph topology and node labeling conventions, and an error is thrown if any node is incompatible or duplicates exist. In other words, the `node_list` must specify a subgraph of the full-yield graph described below. An exception is allowed if `check_edge_list=False`, in which case any node in `edge_list` is treated as valid.
- **check_edge_list** (bool (optional, default `False`)) – If `True`, the `edge_list` elements are checked for compatibility with the graph topology and node labeling conventions, an error is thrown if any edge is incompatible or duplicates exist. In other words, the `edge_list` must specify a subgraph of the full-yield graph described below.

Returns **G** – An (m, n, t) Chimera lattice. Nodes are labeled by integers.

Return type NetworkX Graph

A Chimera lattice is an m-by-n grid of Chimera tiles. Each Chimera tile is itself a bipartite graph with shores of size t. The connection in a Chimera lattice can be expressed using a node-indexing notation (i, j, u, k) for each node.

- (i, j) indexes the (row, column) of the Chimera tile. i must be between 0 and m - 1, inclusive, and j must be between 0 and n - 1, inclusive.
- u=0 indicates the left-hand nodes in the tile, and u=1 indicates the right-hand nodes.
- k=0, 1, . . . , t - 1 indexes nodes within either the left- or right-hand shores of a tile.

In this notation, two nodes (i, j, u, k) and (i', j', u', k') are neighbors if and only if:

(i = i' AND j = j' AND u != u') OR (i = i' +/- 1 AND j = j' AND u = 0 AND u' = 0 AND k = k') OR (i = i' AND j = j' +/- 1 AND u = 1 AND u' = 1 AND k = k')

The first of the three terms of the disjunction gives the bipartite connections within the tile. The second and third terms give the vertical and horizontal connections between blocks respectively.

Node (i, j, u, k) is labeled by:

label = i * n * 2 * t + j * 2 * t + u * t + k

Examples

```
>>> G = dnx.chimera_graph(1, 1, 2)  # a single Chimera tile
>>> len(G)
4
>>> list(G.nodes())  # doctest: +SKIP
[0, 1, 2, 3]
>>> list(G.nodes(data=True))  # doctest: +SKIP
[(0, {'chimera_index': (0, 0, 0, 0)}),
 (1, {'chimera_index': (0, 0, 0, 1)}),
 (2, {'chimera_index': (0, 0, 1, 0)}),
 (3, {'chimera_index': (0, 0, 1, 1)})]
>>> list(G.edges())  # doctest: +SKIP
[(0, 2), (0, 3), (1, 2), (1, 3)]
```

dwave_networkx.pegasus_graph

pegasus_graph(*m*, *create_using=None*, *node_list=None*, *edge_list=None*, *data=True*, *offset_lists=None*, *offsets_index=None*, *coordinates=False*, *fabric_only=True*, *nice_coordinates=False*, *check_node_list=False*, *check_edge_list=False*)

Creates a Pegasus graph with size parameter *m*.

Parameters

- **m** (*int*) – Size parameter for the Pegasus lattice.
- **create_using** (*Graph, optional (default None)*) – If provided, this graph is cleared of nodes and edges and filled with the new graph. Usually used to set the type of the graph.
- **node_list** (*iterable (optional, default None)*) – Iterable of nodes in the graph. The nodes should typically be compatible with the requested lattice shape parameters and coordinate system, incompatible nodes are accepted unless you set `check_node_list=True`. If not specified, calculated from `m`, `fabric_only`, `nice_coordinates`, `offset_lists` and `offset_index` and `coordinates` per the topology description below.

- **edge_list** (*iterable (optional, default None)*) – Iterable of edges in the graph. Edges must be 2-tuples of the nodes specified in `node_list`, or calculated from `m`, `fabric_only`, `nice_coordinates`, `offset_lists` and `offset_index` and `coordinates` per the topology description below; incompatible edges are ignored unless you set `check_edge_list=True`. If not specified, all edges compatible with the `node_list` and topology description are included.
- **data** (bool, optional (default `True`)) – If `True`, each node has a pegasus_index attribute. The attribute is a 4-tuple Pegasus index as defined below. If the *coordinates* parameter is `True`, a linear_index, which is an integer, is used.
- **coordinates** (bool, optional (default `False`)) – If `True`, node labels are 4-tuple Pegasus indices. Ignored if the *nice_coordinates* parameter is `True`.
- **offset_lists** (*pair of lists, optional (default None)*) – Directly controls the offsets. Each list in the pair must have length 12 and contain even ints. If *offset_lists* is not None, the *offsets_index* parameter must be None.
- **offsets_index** (*int, optional (default None)*) – A number between 0 and 7, inclusive, that selects a preconfigured set of topological parameters. If both the *offsets_index* and *offset_lists* parameters are None, the *offsets_index* parameters is set to zero. At least one of these two parameters must be None.
- **fabric_only** (bool, optional (default `True`)) – The Pegasus graph, by definition, has some disconnected components. If `True`, the generator only constructs nodes from the largest component. If `False`, the full disconnected graph is constructed. Ignored if the *edge_lists* parameter is not None or *nice_coordinates* is `True`
- **nice_coordinates** (bool, optional (default `False`)) – If the *offsets_index* parameter is 0, the graph uses a "nicer" coordinate system, more compatible with Chimera addressing. These coordinates are 5-tuples taking the form (t, y, x, u, k) where $0 <= x < M - 1$, $0 <= y < M - 1$, $0 <= u < 2$, $0 <= k < 4$, and $0 <= t < 3$. For any given $0 <= t0 < 3$, the subgraph of nodes with $t = t0$ has the structure of *chimera(M-1, M-1, 4)* with the addition of odd couplers. Supercedes both the *fabric_only* and *coordinates* parameters.
- **check_node_list** (bool (optional, default `False`)) – If `True`, the `node_list` elements are checked for compatibility with the graph topology and node labeling conventions, an error is thrown if any node is incompatible or duplicates exist. In other words, only node lists that specify subgraphs of the default (full yield) graph are permitted. An exception is allowed if `check_edge_list=False`, in which case any node in `edge_list` is treated as valid.
- **check_edge_list** (bool (optional, default `False`)) – If `True`, the edge_list elements are checked for compatibility with the graph topology and node labeling conventions, an error is thrown if any edge is incompatible or duplicates exist. In other words, only edge_lists that specify subgraphs of the default (full yield) graph are permitted.

Returns **G** – A Pegasus lattice for size parameter *m*.

Return type NetworkX Graph

The maximum degree of this graph is 15. The number of nodes depends on multiple parameters; for example,

- `pegasus_graph(1)`: zero nodes
- `pegasus_graph(m, fabric_only=False)`: $24m(m-1)$ nodes
- `pegasus_graph(m, fabric_only=True)`: $24m(m-1) - 8(m-1)$ nodes
- `pegasus_graph(m, nice_coordinates=True)`: $24(m-1)^2$ nodes

Counting formulas for edges have a complicated dependency on parameter settings. Some example upper bounds are:

- `pegasus_graph(1, fabric_only=False)`: zero edges
- `pegasus_graph(m, fabric_only=False)`: $12 * (15 * (m-1)^2 + m - 3)$ edges if $m > 1$

Note that the formulas above are valid for default offset parameters.

A Pegasus lattice is a graph minor of a lattice similar to Chimera, where unit tiles are completely connected. In its most general definition, prelattice $Q(N0, N1)$ contains nodes of the form

- vertical nodes: $(i, j, 0, k)$ with $0 <= k < 2$
- horizontal nodes: $(i, j, 1, k)$ with $0 <= k < 2$

for $0 <= i <= N0$ and $0 <= j < N1$, and edges of the form

- external: (i, j, u, k) ~ $(i + u, j + 1 - u, u, k)$
- internal: $(i, j, 0, k)$ ~ $(i, j, 1, h)$
- odd: $(i, j, u, 0)$ ~ $(i, j, u, 1)$

Given two lists of offsets, $S0$ and $S1$, of length $L0$ and $L1$, where both lengths and values must be divisible by 2, the minor—a Pegasus lattice—is constructed by contracting the complete intervals of external edges:

```
I(0, w, k, z) = [(L1*w + k, L0*z + S0[k] + r, 0, k % 2) for 0 <= r < L0]
I(1, w, k, z) = [(L1*z + S1[k] + r, L0*w + k, 1, k % 2) for 0 <= r < L1]
```

and deleting the prelattice nodes of any interval not fully contained in $Q(N0, N1)$.

This generator, 'pegasus_graph()', is specialized for the minor constructed by prelattice and offset parameters $L0 = L1 = 12$ and $N0 = N1 = 12m$.

The *Pegasus index* of a node in a Pegasus lattice, (u, w, k, z), can be interpreted as:

- u: qubit orientation (0 = vertical, 1 = horizontal)
- w: orthogonal major offset
- k: orthogonal minor offset
- z: parallel offset

Edges in the minor have the form

- external: (u, w, k, z) ~ $(u, w, k, z + 1)$
- internal: $(0, w0, k0, z0)$ ~ $(1, w1, k1, z1)$
- odd: $(u, w, 2k, z)$ ~ $(u, w, 2k + 1, z)$

where internal edges only exist when

1. w1 = z0 + (1 if k1 < S0[k0] else 0)
2. z1 = w0 - (1 if k0 < S1[k1] else 0)

Linear indices are computed from Pegasus indices by the formula:

```
q = ((u * m + w) * 12 + k) * (m - 1) + z
```

Examples

```
>>> G = dnx.pegasus_graph(2, nice_coordinates=True)
>>> G.nodes(data=True)[(0, 0, 0, 0, 0)]     # doctest: +SKIP
{'linear_index': 4, 'pegasus_index': (0, 0, 4, 0)}
```

dwave_networkx.zephyr_graph

zephyr_graph(*m*, *t=4*, *create_using=None*, *node_list=None*, *edge_list=None*, *data=True*, *coordinates=False*, *check_node_list=False*, *check_edge_list=False*)

Creates a Zephyr graph with grid parameter `m` and tile parameter `t`.

The Zephyr topology is described in [BRK].

Parameters

- **m** (*int*) – Grid parameter for the Zephyr lattice.
- **t** (*int*) – Tile parameter for the Zephyr lattice.
- **create_using** (*Graph, optional (default None)*) – If provided, this graph is cleared of nodes and edges and filled with the new graph. Usually used to set the type of the graph.
- **node_list** (*iterable (optional, default None)*) – Iterable of nodes in the graph. If not specified, calculated from (`m`, `t`) and `coordinates`. The nodes should typically be compatible with the requested lattice shape parameters and coordinate system, incompatible nodes are accepted unless you set `check_node_list=True`. If not specified, all $4tm(2m+1)$ nodes compatible with the topology description are included.
- **edge_list** (*iterable (optional, default None)*) – Iterable of edges in the graph. Edges must be 2-tuples of the nodes specified in node_list, or calculated from (`m`, `t`) and `coordinates` per the topology description below; incompatible edges are ignored unless you set `check_edge_list=True`. If not specified, all edges compatible with the `node_list` and topology description are included.
- **data** (bool, optional (default `True`)) – If `True`, adds to each node an attribute with a format that depends on the `coordinates` parameter: a 5-tuple `'zephyr_index'` if `coordinates=False` and an integer `'linear_index'` if `coordinates` is `True`.
- **coordinates** (bool, optional (default `False`)) – If `True`, node labels are 5-tuple Zephyr indices.
- **check_node_list** (bool (optional, default `False`)) – If `True`, the `node_list` elements are checked for compatibility with the graph topology and node labeling conventions, and an error is thrown if any node is incompatible or duplicates exist. In other words, `node_lists` must specify a subgraph of the default (full yield) graph described below. An exception is allowed if `check_edge_list=False`, any node in edge_list will also be treated as valid.
- **check_edge_list** (bool (optional, default `False`)) – If `True`, `edge_list` elements are checked for compatibility with the graph topology and node labeling conventions, and an error is thrown if any edge is incompatible or duplicates exist. In other words, `edge_list` must specify a subgraph of the default (full yield) graph described below.

Returns **G** – A Zephyr lattice for grid parameter `m` and tile parameter `t`.

Return type NetworkX Graph

The maximum degree of this graph is $4t+4$. The number of nodes is given by

- `zephyr_graph(m, t)`: $4tm(2m+1)$

The number of edges depends on parameter settings,

- `zephyr_graph(1, t)`: $2t(8t+3)$
- `zephyr_graph(m, t)`: $2t((8t+8)m^2 - 2m - 3)$ if m > 1

A Zephyr lattice is a graph minor of a lattice similar to Chimera, where unit tiles have odd couplers similar to Pegasus graphs. In its most general definition, prelattice $Q(2m+1)$ contains nodes of the form

- vertical nodes: $(i, j, 0, k)$ with $0 <= k < 2t$
- horizontal nodes: $(i, j, 1, k)$ with $0 <= k < 2t$

for $0 <= i < 2m+1$ and $0 <= j < 2m+1$, and edges of the form

- external: (i, j, u, k) ~ $(i+u, j+1-u, u, k)$
- internal: $(i, j, 0, k)$ ~ $(i, j, 1, h)$
- odd: $(i, j, u, 2k)$ ~ $(i, j, u, 2k+1)$

The minor—a Zephyr lattice—is constructed by contracting pairs of external edges:

```
I(0, w, k, j, z) = [(2*z+j, w, 0, 2*k+j), (2*z+1+j, w, 0, 2*k+j)]
I(1, w, k, j, z) = [(w, 2*z+j, 1, 2*k+j), (w, 2*z+1+j, 1, 2*k+j)]
```

and deleting the prelattice nodes of any pair not fully contained in $Q(2m+1)$.

The *Zephyr index* of a node in a Zephyr lattice, (u, w, k, j, z), can be interpreted as:

- u: qubit orientation (0 = vertical, 1 = horizontal)
- w: orthogonal major offset; $0 <= w < 2m+1$
- k: orthogonal secondary offset; $0 <= k < t$
- j: orthogonal minor offset; $0 <= j < 2$
- z: parallel offset; $0 <= z < m$

Edges in the minor have the form

- external: (u, w, k, j, z) ~ $(u, w, k, j, z+1)$
- odd: $(u, w, 2k, z)$ ~ $(u, w, 2k+1, z-a)$
- internal: $(0, 2w+1-a, k, j, z-jb)$ ~ $(1, 2z+1-b, h, i, w-ia)$

for $0 <= a < 2$ and $0 <= b < 2$, where internal edges only exist when

1. $0 <= 2w+1-a < 2m+1$,
2. $0 <= 2z+1-a < 2m+1$,
3. $0 <= z-jb < m$, and
4. $0 <= w-ia < m$.

Linear indices are computed from Zephyr indices by the formula:

```
q = (((u * (2 * m + 1) + w) * t + k) * 2 + j) * m + z
```

Examples

```
>>> G = dnx.zephyr_graph(2)
>>> G.nodes(data=True)[(0, 0, 0, 0, 0)]     # doctest: +SKIP
{'linear_index': 0}
```

References

Example

This example uses the the *chimera_graph()* function to create a Chimera lattice of size (1, 1, 4), which is a single unit cell in Chimera topology, and the *find_chimera()* function to determine the Chimera indices.

```
>>> import networkx as nx
>>> import dwave_networkx as dnx
>>> G = dnx.chimera_graph(1, 1, 4)
>>> chimera_indices = dnx.find_chimera_indices(G)
>>> print chimera_indices
{0: (0, 0, 0, 0),
 1: (0, 0, 0, 1),
 2: (0, 0, 0, 2),
 3: (0, 0, 0, 3),
 4: (0, 0, 1, 0),
 5: (0, 0, 1, 1),
 6: (0, 0, 1, 2),
 7: (0, 0, 1, 3)}
```

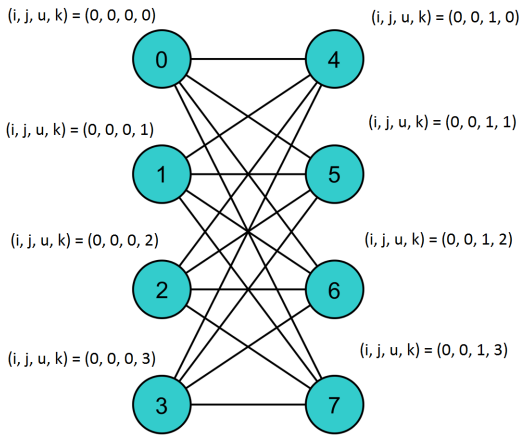

Fig. 12: Indices of a Chimera unit cell found by creating a lattice of size (1, 1, 4).

Toruses

chimera_torus(m[, n, t, node_list, edge_list])	Creates a defect-free Chimera lattice of size (m, n, t) subject to periodic boundary conditions.

Continued on next page

Table 18 – continued from previous page

pegasus_torus(m[, node_list, edge_list, ...])	Creates a Pegasus graph modified to allow for periodic boundary conditions and translational invariance.
zephyr_torus(m[, t, node_list, edge_list])	Creates a Zephyr graph modified to allow for periodic boundary conditions and translational invariance.

dwave_networkx.chimera_torus

chimera_torus(*m*, *n=None*, *t=None*, *node_list=None*, *edge_list=None*)

Creates a defect-free Chimera lattice of size (m, n, t) subject to periodic boundary conditions.

Parameters

- **m** (*int*) – Number of rows in the Chimera torus lattice. If $m < 3$ translational invariance already applies in the rows. If $m >= 3$ additional external couplers are added, reestablishing translational invariance. Connectivity of all horizontal qubits is $min(m - 1, 2) + 2t$.
- **n** (*int (optional, default m)*) – Number of columns in the Chimera torus lattice. If $n < 3$ translational invariance already applies in the columns. If $n >= 3$ additional external couplers are added, reestablishing translational invariance. Connectivity of all vertical qubits is $min(n - 1, 2) + 2t$.
- **t** (*int (optional, default 4)*) – Size of the shore within each Chimera tile.
- **node_list** (*iterable (optional, default None)*) – Iterable of nodes in the graph. If None, nodes are generated for an undiluted torus calculated from `m`, `n` and `t` as described below. The node list must describe a subset of the torus nodes to be maintained in the graph using the coordinate node labeling scheme.
- **edge_list** (*iterable (optional, default None)*) – Iterable of edges in the graph. If None, edges are generated for an undiluted torus calculated from `m`, `n` and `t` as described below. The edge list must describe a subgraph of the torus, using the coordinate node labeling scheme.

Returns **G** – A Chimera torus with shape (m, n, t), with Chimera coordinate node labels.

Return type NetworkX Graph

A Chimera torus is a generalization of the standard chimera graph whereby degree-six connectivity is maintained, but the boundary condition is modified to enforce an additional translational-invariance symmetry [RH]. Local connectivity in the Chimera torus is identical to connectivity for chimera graph nodes away from the boundary. The graph has `V=8*m*n` nodes, and `min(6, 4 + m)V//2 + min(6, 4 + n)V/2` edges. With the standard $K_{t,t}$ Chimera tile definition, any tile displacement (x, y) modulo (m, n), rows and columns respectively, that is, `(i, j, u, k) -> ((i + x)%m, (i + y)%n, u, k)`, defines an automorphism.

See *chimera_graph()* for additional information.

Examples

```
>>> G = dnx.chimera_torus(3, 3, 4)  # a 3x3 tile chimera graph (connectivity 6)
>>> len(G)
72
>>> any([len(list(G.neighbors(n))) != 6 for n in G.nodes])
False
```

dwave_networkx.pegasus_torus

pegasus_torus(*m*, *node_list=None*, *edge_list=None*, *offset_lists=None*, *offsets_index=None*)

Creates a Pegasus graph modified to allow for periodic boundary conditions and translational invariance.

Parameters

- **m** (*int*) – Size parameter for the Pegasus lattice. Connectivity of all nodes is $13+min(m-1, 2)$
- **node_list** (*iterable (optional, default None)*) – Iterable of nodes in the graph. If None, nodes are generated for an undiluted torus calculated from `m` as described below. The node list must describe a subset of the torus nodes to be maintained in the graph using the coordinate node labeling scheme.
- **edge_list** (*iterable (optional, default None)*) – Iterable of edges in the graph. If None, edges are generated for an undiluted torus calculated from `m` as described below. The edge list must describe a subgraph of the torus, using the coordinate node labeling scheme.
- **offset_lists** (*pair of lists, optional (default None)*) – Directly controls the offsets. Each list in the pair must have length 12 and contain even integers. If `offset_lists` is not None, the `offsets_index` parameter must be None.
- **offsets_index** (*int, optional (default None)*) – A number between 0 and 7, inclusive, that selects a preconfigured set of topological parameters. If both the `offsets_index` and `offset_lists` parameters are None, the `offsets_index` parameters is set to zero. At least one of these two parameters must be None.

Returns **G** – A Pegasus torus for size parameter m using the coordinate labeling system.

Return type NetworkX Graph

A Pegasus torus is a generalization of the standard Pegasus graph whereby degree-fifteen connectivity is maintained, but the boundary condition is modified to enforce an additional translational-invariance symmetry [RH]. Local connectivity in the Pegasus torus is identical to connectivity for Pegasus graph nodes away from the boundary. A tile consists of 24 nodes, and the torus has $m-1$ by $m-1$ tiles. Tile displacement modulo $m-1$ defines an automorphism.

See *pegasus_graph()* for additional information.

Examples

```
>>> G = dnx.pegasus_torus(4)  # a 3x3 tile pegasus torus (connectivity 15)
>>> len(G) # 3*3*24
216
>>> any([len(list(G.neighbors(n))) != 15 for n in G.nodes])
False
```

dwave_networkx.zephyr_torus

zephyr_torus(*m*, *t=4*, *node_list=None*, *edge_list=None*)

Creates a Zephyr graph modified to allow for periodic boundary conditions and translational invariance.

The graph matches the local connectivity properties of a standard Zephyr graph, but with modified periodic boundary condition. Tiles of $8t$ nodes are arranged on an m by m torus.

Parameters

- **m** (*int*) – Grid parameter for the Zephyr lattice. Connectivity of all nodes is $4t+min(2m-1, 4)$.
- **t** (*int*) – Tile parameter for the Zephyr lattice.
- **node_list** (*iterable (optional, default None)*) – Iterable of nodes in the graph. If None, nodes are generated for an undiluted torus calculated from `m` and `t` as described below. The node list must describe a subset of the torus nodes to be maintained in the graph using the coordinate node labeling scheme.
- **edge_list** (*iterable (optional, default None)*) – Iterable of edges in the graph. If None, edges are generated for an undiluted torus calculated from `m` and `t` as described below. The edge list must describe a subgraph of the torus, using the coordinate node labeling scheme.

Returns **G** – A Zephyr torus with grid parameter `m` and tile parameter `t`, with Zephyr coordinate node labels.

Return type NetworkX Graph

A Zephyr torus is a generalization of the standard Zephyr graph whereby degree-twenty connectivity is maintained, but the boundary condition is modified to enforce an additional translational-invariance symmetry [RH]. Local connectivity in the Zephyr torus is identical to connectivity for Zephyr graph nodes away from the boundary. A tile consists of $8t$ nodes, and the torus has m by m tiles. Tile displacement modulo m defines an automorphism.

See *zephyr_graph()* for additional information.

Examples

```
>>> G = dnx.zephyr_torus(3)  # a 3x3 tile pegasus torus (connectivity 15)
>>> len(G) # 3*3*24
288
>>> any([len(list(G.neighbors(n))) != 20 for n in G.nodes])
False
```

Other Graphs

markov_network(potentials)	Creates a Markov Network from potentials.

dwave_networkx.markov_network

markov_network (*potentials*)

Creates a Markov Network from potentials.

A Markov Network is also knows as a Markov Random Field

Parameters **potentials** (*dict[tuple, dict]*) – A dict where the keys are either nodes or edges and the values are a dictionary of potentials. The potential dict should map each possible assignment of the nodes/edges to their energy.

Returns **MN** – A markov network as a graph where each node/edge stores its potential dict as above.

Return type networkx.Graph

Examples

```
>>> potentials = {('a', 'b'): {(0, 0): -1,
...                            (0, 1): .5,
...                            (1, 0): .5,
...                            (1, 1): 2}}
>>> MN = dnx.markov_network(potentials)
>>> MN['a']['b']['potential'][(0, 0)]
-1
```

Utilities

Decorators

Decorators allow for input checking and default parameter setting for algorithms.

binary_quadratic_model_sampler(which_args)	Decorator to validate sampler arguments.

dwave_networkx.utils.decorators.binary_quadratic_model_sampler

binary_quadratic_model_sampler(*which_args*)

Decorator to validate sampler arguments.

Parameters **which_args** (*int or sequence of ints*) – Location of the sampler arguments of the input function in the form *function_name(args, *kw)*. If more than one sampler is allowed, can be a list of locations.

Coordinates Conversion

class chimera_coordinates(*m*, *n=None*, *t=None*)

Provides coordinate converters for the chimera indexing scheme.

Parameters

- **m** (*int*) – The number of rows in the Chimera lattice.
- **n** (*int, optional (default m)*) – The number of columns in the Chimera lattice.
- **t** (*int, optional (default 4)*) – The size of the shore within each Chimera tile.

Examples

Convert between Chimera coordinates and linear indices directly

```
>>> coords = dnx.chimera_coordinates(16, 16, 4)
>>> coords.chimera_to_linear((0, 2, 0, 1))
17
>>> coords.linear_to_chimera(17)
(0, 2, 0, 1)
```

Construct a new graph with the coordinate labels

```
>>> C16 = dnx.chimera_graph(16)
>>> coords = dnx.chimera_coordinates(16)
>>> G = nx.Graph()
>>> G.add_nodes_from(coords.iter_linear_to_chimera(C16.nodes))
>>> G.add_edges_from(coords.iter_linear_to_chimera_pairs(C16.edges))
```

See also:

chimera_graph() Describes the various conventions.

class pegasus_coordinates(*m*)

Provides coordinate converters for the Pegasus indexing schemes.

Parameters **m** (*int*) – Size parameter for the Pegasus lattice.

See also:

pegasus_graph() Describes the various coordinate conventions.

class zephyr_coordinates(*m*, *t=4*)

Provides coordinate converters for the Zephyr indexing schemes.

Parameters

- **m** (*int*) – Grid parameter for the Zephyr lattice.
- **t** (*int*) – Tile parameter for the Zephyr lattice; must be even.

See also:

zephyr_graph() Describes the various coordinate conventions.

Graph Indexing

See *Coordinates Conversion* on instantiating the needed lattice size and setting correct domain and range for coordinates in a QPU working graph.

For the iterator versions of these functions, see the code.

Chimera

chimera_coordinates.chimera_to_linear(q)	Convert a 4-term Chimera coordinate to a linear index.
chimera_coordinates.linear_to_chimera(r)	Convert a linear index to a 4-term Chimera coordinate.
find_chimera_indices(G)	Attempts to determine the Chimera indices of the nodes in graph G.

dwave_networkx.chimera_coordinates.chimera_to_linear

chimera_coordinates.**chimera_to_linear**(*q*)

Convert a 4-term Chimera coordinate to a linear index.

Parameters **q** (*4-tuple*) – Chimera coordinate.

Examples

```
>>> dnx.chimera_coordinates(16).chimera_to_linear((2, 2, 0, 0))
272
```

dwave_networkx.chimera_coordinates.linear_to_chimera

chimera_coordinates.**linear_to_chimera**(*r*)

Convert a linear index to a 4-term Chimera coordinate.

Parameters **r** (*int*) – Linear index.

Examples

```
>>> dnx.chimera_coordinates(16).linear_to_chimera(272)
(2, 2, 0, 0)
```

dwave_networkx.find_chimera_indices

find_chimera_indices(*G*)

Attempts to determine the Chimera indices of the nodes in graph G.

See the *chimera_graph()* function for a definition of a Chimera graph and Chimera indices.

Parameters **G** (*NetworkX graph*) – Should be a single-tile Chimera graph.

Returns **chimera_indices** – A dict of the form {node: (i, j, u, k), ... } where (i, j, u, k) is a 4-tuple of integer Chimera indices.

Return type dict

Examples

```
>>> G = dnx.chimera_graph(1, 1, 4)
>>> chimera_indices = dnx.find_chimera_indices(G)
```

```
>>> G = nx.Graph()
>>> G.add_edges_from([(0, 2), (1, 2), (1, 3), (0, 3)])
>>> chimera_indices = dnx.find_chimera_indices(G)
>>> nx.set_node_attributes(G, chimera_indices, 'chimera_index')
```

Pegasus

pegasus_coordinates.linear_to_nice(r)	Convert a linear index into a 5-term nice coordinate.
pegasus_coordinates.linear_to_pegasus(r)	Convert a linear index into a 4-term Pegasus coordinate.
pegasus_coordinates.nice_to_linear(n)	Convert a 5-term nice coordinate into a linear index.

Continued on next page

Table 22 – continued from previous page

pegasus_coordinates.nice_to_pegasus(n)	Convert a 5-term nice coordinate into a 4-term Pegasus coordinate.
pegasus_coordinates.pegasus_to_linear(q)	Convert a 4-term Pegasus coordinate into a linear index.
pegasus_coordinates.pegasus_to_nice(p)	Convert a 4-term Pegasus coordinate to a 5-term nice coordinate.

dwave_networkx.pegasus_coordinates.linear_to_nice

pegasus_coordinates.**linear_to_nice**(*r*)

Convert a linear index into a 5-term nice coordinate.

Parameters **r** (*int*) – Linear index.

Examples

```
>>> dnx.pegasus_coordinates(2).linear_to_nice(4)
(0, 0, 0, 0, 0)
```

dwave_networkx.pegasus_coordinates.linear_to_pegasus

pegasus_coordinates.**linear_to_pegasus**(*r*)

Convert a linear index into a 4-term Pegasus coordinate.

Parameters **r** (*int*) – Linear index.

Examples

```
>>> dnx.pegasus_coordinates(2).linear_to_pegasus(4)
(0, 0, 4, 0)
```

dwave_networkx.pegasus_coordinates.nice_to_linear

pegasus_coordinates.**nice_to_linear**(*n*)

Convert a 5-term nice coordinate into a linear index.

Parameters **n** (*5-tuple*) – Nice coordinate.

Examples

```
>>> dnx.pegasus_coordinates(2).nice_to_linear((0, 0, 0, 0, 0))
4
```

dwave_networkx.pegasus_coordinates.nice_to_pegasus

static pegasus_coordinates.**nice_to_pegasus**(*n*)

Convert a 5-term nice coordinate into a 4-term Pegasus coordinate.

Parameters **n** (*5-tuple*) – Nice coordinate.

Examples

```
>>> dnx.pegasus_coordinates.nice_to_pegasus((0, 0, 0, 0, 0))
(0, 0, 4, 0)
```

Note that this method does not depend on the size of the Pegasus lattice.

dwave_networkx.pegasus_coordinates.pegasus_to_linear

pegasus_coordinates.**pegasus_to_linear**(*q*)
Convert a 4-term Pegasus coordinate into a linear index.

Parameters **q** (*4-tuple*) – Pegasus indices.

Examples

```
>>> dnx.pegasus_coordinates(2).pegasus_to_linear((0, 0, 4, 0))
4
```

dwave_networkx.pegasus_coordinates.pegasus_to_nice

static pegasus_coordinates.**pegasus_to_nice**(*p*)
Convert a 4-term Pegasus coordinate to a 5-term nice coordinate.

Parameters **p** (*4-tuple*) – Pegasus coordinate.

Examples

```
>>> dnx.pegasus_coordinates.pegasus_to_nice((0, 0, 4, 0))
(0, 0, 0, 0, 0)
```

Note that this method does not depend on the size of the Pegasus lattice.

Zephyr

zephyr_coordinates.graph_to_linear(g)	Return a copy of the graph g relabeled to have linear indices
zephyr_coordinates.graph_to_zephyr(g)	Return a copy of the graph g relabeled to have zephyr coordinates
zephyr_coordinates.iter_linear_to_zephyr(rlist)	Return an iterator converting a sequence of linear indices to 5-term Zephyr coordinates.
zephyr_coordinates.iter_linear_to_zephyr_pairs(plist)	Return an iterator converting a sequence of pairs of linear indices to pairs of 5-term Zephyr coordinates.
zephyr_coordinates.iter_zephyr_to_linear(qlist)	Return an iterator converting a sequence of 5-term Zephyr coordinates to linear indices.

Continued on next page

Table 23 – continued from previous page

zephyr_coordinates.iter_zephyr_to_linear_pairs(plist)	Return an iterator converting a sequence of pairs of 5-term Zephyr coordinates to pairs of linear indices.
zephyr_coordinates.linear_to_zephyr(r)	Convert a linear index into a 5-term Zephyr coordinate.
zephyr_coordinates.zephyr_to_linear(q)	Convert a 5-term Zephyr coordinate into a linear index.
zephyr_sublattice_mappings(source, target[, ...])	Yields mappings from a Chimera or Zephyr graph into a Zephyr graph.

dwave_networkx.zephyr_coordinates.graph_to_linear

zephyr_coordinates.**graph_to_linear**(*g*)
Return a copy of the graph g relabeled to have linear indices

dwave_networkx.zephyr_coordinates.graph_to_zephyr

zephyr_coordinates.**graph_to_zephyr**(*g*)
Return a copy of the graph g relabeled to have zephyr coordinates

dwave_networkx.zephyr_coordinates.iter_linear_to_zephyr

zephyr_coordinates.**iter_linear_to_zephyr**(*rlist*)
Return an iterator converting a sequence of linear indices to 5-term Zephyr coordinates.

dwave_networkx.zephyr_coordinates.iter_linear_to_zephyr_pairs

zephyr_coordinates.**iter_linear_to_zephyr_pairs**(*plist*)
Return an iterator converting a sequence of pairs of linear indices to pairs of 5-term Zephyr coordinates.

dwave_networkx.zephyr_coordinates.iter_zephyr_to_linear

zephyr_coordinates.**iter_zephyr_to_linear**(*qlist*)
Return an iterator converting a sequence of 5-term Zephyr coordinates to linear indices.

dwave_networkx.zephyr_coordinates.iter_zephyr_to_linear_pairs

zephyr_coordinates.**iter_zephyr_to_linear_pairs**(*plist*)
Return an iterator converting a sequence of pairs of 5-term Zephyr coordinates to pairs of linear indices.

dwave_networkx.zephyr_coordinates.linear_to_zephyr

zephyr_coordinates.**linear_to_zephyr**(*r*)
Convert a linear index into a 5-term Zephyr coordinate.

Parameters **r** (*int*) – Linear index.

Examples

```
>>> dnx.zephyr_coordinates(2).linear_to_zephyr(137)
(1, 3, 2, 0, 1)
```

dwave_networkx.zephyr_coordinates.zephyr_to_linear

zephyr_coordinates.**zephyr_to_linear**(*q*)

Convert a 5-term Zephyr coordinate into a linear index.

Parameters **q** (*5-tuple*) – Zephyr coordinate.

Examples

```
>>> dnx.zephyr_coordinates(2).zephyr_to_linear((0, 1, 2, 1, 0))
26
```

dwave_networkx.zephyr_sublattice_mappings

zephyr_sublattice_mappings(*source*, *target*, *offset_list=None*)

Yields mappings from a Chimera or Zephyr graph into a Zephyr graph.

A sublattice mapping is a function from nodes of

- a `zephyr_graph(m_s, t)` to nodes of a `zephyr_graph(m_t, t)` where `m_s <= m_t`,
- a `chimera_graph(m_s, n_s, t)` to nodes of a `zephyr_graph(m_t, t)` where `m_s <= 2*m_t` and `n_s <= 2*m_t`, or
- a `chimera_graph(m_s, n_s, 2*t)` to nodes of a `zephyr_graph(m_t, t)` where `m_s <= m_t` and `n_s <= m_t`, or

This is used to identify subgraphs of the target Zephyr graphs which are isomorphic to the source graph. However, if the target graph is not of perfect yield, these functions do not generally produce isomorphisms (for example, if a node is missing in the target graph, it may still appear in the image of the source graph).

Note that the tile parameter of Chimera graphs must be either the same or double that of the target Zephyr graphs; if both graphs are Zephyr graphs, the tile parameters must be the same. The mappings produced preserve the linear ordering of tile indices; see the `_zephyr_zephyr_sublattice_mapping`, `_double_chimera_zephyr_sublattice_mapping`, and `_single_chimera_zephyr_sublattice_mapping` internal functions for more details.

Academic note: the full group of isomorphisms of a Chimera graph includes mappings which permute tile indices on a per-row and per-column basis, in addition to reflections and rotations of the grid of unit tiles where rotations by 90 and 270 degrees induce a change in orientation. The isomorphisms of Zephyr graphs permit permutations of major tile indices on a per-row and per-column basis, in addition to reflections of the grid which induce inversion of orthogonal minor offsets, and rotations which induce inversions of minor offsets and/or orientation. The full set of sublattice mappings would take those isomorphisms into account; we do not undertake that complexity here.

Parameters

- **source** (*NetworkX Graph*) – The Chimera or Zephyr graph that nodes are input from.
- **target** (*NetworkX Graph*) – The Zephyr graph that nodes are output to.

- **offset_list** (*iterable (tuple), optional (default None)*) – An iterable of offsets. This can be used to reconstruct a set of mappings, as the offset used to generate a single mapping is stored in the `offset` attribute of that mapping.

Yields mapping (*function*) – A function from nodes of the source graph, to nodes of the target graph. The offset used to generate this mapping is stored in `mapping.offset` – these can be collected and passed into `offset_list` in a later session.

Exceptions

Base exceptions and errors for D-Wave NetworkX.

All exceptions are derived from NetworkXException.

DWaveNetworkXException	Base class for exceptions in DWaveNetworkX.
DWaveNetworkXMissingSampler	Exception raised by an algorithm requiring a discrete model sampler when none is provided.

dwave_networkx.exceptions.DWaveNetworkXException

exception DWaveNetworkXException
Base class for exceptions in DWaveNetworkX.

dwave_networkx.exceptions.DWaveNetworkXMissingSampler

exception DWaveNetworkXMissingSampler
Exception raised by an algorithm requiring a discrete model sampler when none is provided.

Default sampler

Sets a binary quadratic model sampler used by default for functions that require a sample when none is specified.

A sampler is a process that samples from low-energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO).

Sampler API

- Required Methods: 'sample_qubo' and 'sample_ising'
- Return value: iterable of samples, in order of increasing energy

See dimod for details.

Example

This example creates and uses a placeholder for binary quadratic model samplers that returns a correct response only in the case of finding an independent set on a complete graph (where one node is always an independent set). The placeholder sampler can be used to test the simple examples of the functions for configuring a default sampler.

```
>>> # Create a placeholder sampler
>>> class ExampleSampler:
...     # an example sampler, only works for independent set on complete
...     # graphs
...     def __init__(self, name):
...         self.name = name
...     def sample_ising(self, h, J):
...         sample = {v: -1 for v in h}
...         sample[0] = 1  # set one node to true
...         return [sample]
...     def sample_qubo(self, Q):
...         sample = {v: 0 for v in set().union(*Q)}
...         sample[0] = 1  # set one node to true
...         return [sample]
...     def __str__(self):
...         return self.name
...
>>> # Identify the new sampler as the default sampler
>>> sampler0 = ExampleSampler('sampler0')
>>> dnx.set_default_sampler(sampler0)
>>> # Find an independent set using the default sampler
>>> G = nx.complete_graph(5)
>>> dnx.maximum_independent_set(G)
[0]
```

Functions

set_default_sampler(sampler)	Sets a default binary quadratic model sampler.
unset_default_sampler()	Resets the default sampler back to None.
get_default_sampler()	Queries the current default sampler.

dwave_networkx.default_sampler.set_default_sampler

set_default_sampler(*sampler*)

Sets a default binary quadratic model sampler.

Parameters **sampler** – A binary quadratic model sampler. A sampler is a process that samples from low-energy states in models defined by an Ising equation or a Quadratic Unconstrained Binary Optimization Problem (QUBO). A sampler is expected to have a 'sample_qubo' and 'sample_ising' method. A sampler is expected to return an iterable of samples, in order of increasing energy.

Examples

This example sets sampler0 as the default sampler and finds an independent set for graph G, first using the default sampler and then overriding it by specifying a different sampler.

```
>>> dnx.set_default_sampler(sampler0)  # doctest: +SKIP
>>> indep_set = dnx.maximum_independent_set_dm(G)  # doctest: +SKIP
>>> indep_set = dnx.maximum_independent_set_dm(G, sampler1)  # doctest: +SKIP
```

dwave_networkx.default_sampler.unset_default_sampler

unset_default_sampler()
Resets the default sampler back to None.

Examples

This example sets sampler0 as the default sampler, verifies the setting, then resets the default, and verifies the resetting.

```
>>> dnx.set_default_sampler(sampler0)  # doctest: +SKIP
>>> print(dnx.get_default_sampler())  # doctest: +SKIP
'sampler0'
>>> dnx.unset_default_sampler()  # doctest: +SKIP
>>> print(dnx.get_default_sampler())  # doctest: +SKIP
None
```

dwave_networkx.default_sampler.get_default_sampler

get_default_sampler()
Queries the current default sampler.

Examples

This example queries the default sampler before and after specifying a default sampler.

```
>>> print(dnx.get_default_sampler())  # doctest: +SKIP
None
>>> dnx.set_default_sampler(sampler)  # doctest: +SKIP
>>> print(dnx.get_default_sampler())  # doctest: +SKIP
'sampler'
```

3.1.3 Bibliography

3.1.4 Installation

Installation from PyPi:

```
pip install dwave_networkx
```

Installation from source:

```
pip install -r requirements.txt
python setup.py install
```

3.1.5 License

Apache License

Version 2.0, January 2004

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3.2 Indices and tables

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