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# **dwavebinarycsp**

*Release 0.2.0*

**D-Wave Systems Inc**

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Library to construct a binary quadratic model from a constraint satisfaction problem with small constraints over binary variables.

Below is an example usage:

```
import dwavebinarycsp
import dimod

csp = dwavebinarycsp.factories.random_2in4sat(8, 4) # 8 variables, 4 clauses

bqm = dwavebinarycsp.stitch(csp)

resp = dimod.ExactSolver().sample(bqm)

for sample, energy in resp.data(['sample', 'energy']):
    print(sample, csp.check(sample), energy)
```



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**Note:** This documentation is for the latest version of `dwavebinarycsp`. Documentation for the version currently installed by `dwave-ocean-sdk` is here: `dwavebinarycsp`.

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## 1.1 Introduction

*dwavebinarycsp* is a library to construct a binary quadratic model from a constraint satisfaction problem (CSP) with small constraints over binary variables (represented as either binary values  $\{0, 1\}$  or spin values  $\{-1, 1\}$ ).

For an introduction to CSPs, see [Constraints](#).

### 1.1.1 Example: Solving a Map-Coloring CSP

The map-coloring CSP, for example, is to assign a color to each region of a map such that any two regions sharing a border have different colors.

The constraints for the map-coloring problem can be expressed as follows:

- Each region is assigned one color only, of  $C$  possible colors.
- The color assigned to one region cannot be assigned to adjacent regions.

Solving such problems as the map-coloring CSP on a sampler such as the D-Wave system necessitates that the mathematical formulation use binary variables because the solution is implemented physically with qubits, and so must translate to spins  $s_i \in \{-1, +1\}$  or equivalent binary values  $x_i \in \{0, 1\}$ . This means that in formulating the problem by stating it mathematically, you might use unary encoding to represent the  $C$  colors: each region is represented by  $C$  variables, one for each possible color, which is set to value 1 if selected, while the remaining  $C - 1$  variables are 0.

This example finds a solution to the map-coloring problem for a map of Canada using four colors. Canada's 13 provinces are denoted by postal codes:



Fig. 1: Coloring a map of Canada with four colors.

Table 1: Canadian Provinces' Postal Codes

Code	Province	Code	Province
AB	Alberta	BC	British Columbia
MB	Manitoba	NB	New Brunswick
NL	Newfoundland and Labrador	NS	Nova Scotia
NT	Northwest Territories	NU	Nunavut
ON	Ontario	PE	Prince Edward Island
QC	Quebec	SK	Saskatchewan
YT	Yukon		

The workflow for solution is as follows:

1. Formulate the problem as a graph, with provinces represented as nodes and shared borders as edges, using 4 binary variables (one per color) for each province.
2. Create a binary constraint satisfaction problem and add all the needed constraints.
3. Convert to a binary quadratic model.
4. Sample.
5. Plot a valid solution, if such was found.

The following sample code creates a graph of the map with provinces as nodes and shared borders between provinces as edges (e.g., “(‘AB’, ‘BC’)” is an edge representing the shared border between British Columbia and Alberta). It creates a binary constraint satisfaction problem based on two types of constraints:

- `csp.add_constraint(one_color_configurations, variables)` represents the constraint that each node (province) select a single color, as represented by valid configurations `one_color_configurations = {(0, 0, 0, 1), (0, 0, 1, 0), (0, 1, 0, 0), (1, 0, 0, 0)}`
- `csp.add_constraint(not_both_1, variables)` represents the constraint that two nodes (provinces) with a shared edge (border) not both select the same color.

```
import dwavebinarycsp
from dwave.system.samplers import DWaveSampler
from dwave.system.composites import EmbeddingComposite
```

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```

import networkx as nx
import matplotlib.pyplot as plt

# Represent the map as the nodes and edges of a graph
provinces = ['AB', 'BC', 'MB', 'NB', 'NL', 'NS', 'NT', 'NU', 'ON', 'PE', 'QC', 'SK',
↳ 'YT']
neighbors = [('AB', 'BC'), ('AB', 'NT'), ('AB', 'SK'), ('BC', 'NT'), ('BC', 'YT'), (
↳ 'MB', 'NU'),
              ('MB', 'ON'), ('MB', 'SK'), ('NB', 'NS'), ('NB', 'QC'), ('NL', 'QC'), (
↳ 'NT', 'NU'),
              ('NT', 'SK'), ('NT', 'YT'), ('ON', 'QC')]

# Function for the constraint that two nodes with a shared edge not both select one_
↳ color
def not_both_1(v, u):
    return not (v and u)

# Function that plots a returned sample
def plot_map(sample):
    G = nx.Graph()
    G.add_nodes_from(provinces)
    G.add_edges_from(neighbors)
    # Translate from binary to integer color representation
    color_map = {}
    for province in provinces:
        for i in range(colors):
            if sample[province+str(i)]:
                color_map[province] = i
    # Plot the sample with color-coded nodes
    node_colors = [color_map.get(node) for node in G.nodes()]
    nx.draw_circular(G, with_labels=True, node_color=node_colors, node_size=3000,
↳ cmap=plt.cm.rainbow)
    plt.show()

# Valid configurations for the constraint that each node select a single color
one_color_configurations = {(0, 0, 0, 1), (0, 0, 1, 0), (0, 1, 0, 0), (1, 0, 0, 0)}
colors = len(one_color_configurations)

# Create a binary constraint satisfaction problem
csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)

# Add constraint that each node (province) select a single color
for province in provinces:
    variables = [province+str(i) for i in range(colors)]
    csp.add_constraint(one_color_configurations, variables)

# Add constraint that each pair of nodes with a shared edge not both select one color
for neighbor in neighbors:
    v, u = neighbor
    for i in range(colors):
        variables = [v+str(i), u+str(i)]
        csp.add_constraint(not_both_1, variables)

# Convert the binary constraint satisfaction problem to a binary quadratic model
bqm = dwavebinarycsp.stitch(csp)

# Set up a solver using the local system's default D-Wave Cloud Client configuration_
↳ file

```

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```

# and sample 50 times
sampler = EmbeddingComposite(DWaveSampler())           # doctest: +SKIP
response = sampler.sample(bqm, num_reads=50)         # doctest: +SKIP

# Plot the lowest-energy sample if it meets the constraints
sample = next(response.samples())                    # doctest: +SKIP
if not csp.check(sample):                          # doctest: +SKIP
    print("Failed to color map")
else:
    plot_map(sample)

```

The plot shows a solution returned by the D-Wave solver. No provinces sharing a border have the same color.

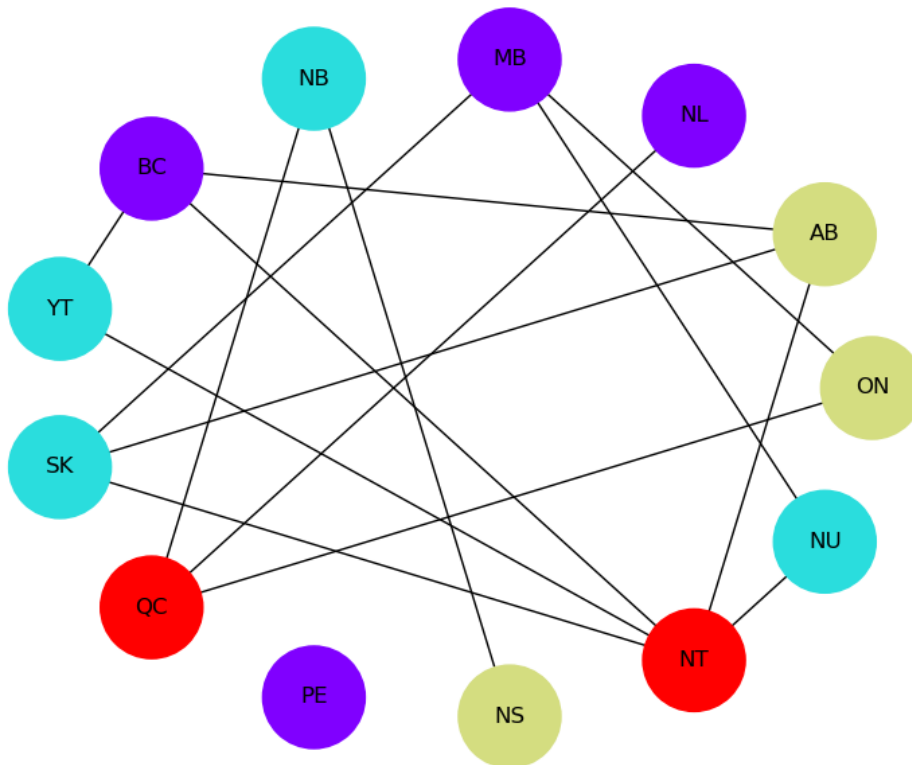


Fig. 2: Solution for a map of Canada with four colors. The graph comprises 13 nodes representing provinces connected by edges representing shared borders. No two nodes connected by an edge share a color.

## 1.2 Reference Documentation

**Release** 0.2.0

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### 1.2.1 Defining Constraint Satisfaction Problems

Constraint satisfaction problems require that all a problem's variables be assigned values, out of a finite domain, that result in the satisfying of all constraints. The `ConstraintSatisfactionProblem` class aggregates all con-

straints and variables defined for a problem and provides functionality to assist in problem solution, such as verifying whether a candidate solution satisfies the constraints.

## Class

**class** `ConstraintSatisfactionProblem` (*vartype*)

A constraint satisfaction problem.

**Parameters** `vartype` (*Vartype*/str/set) – Variable type for the binary quadratic model. Supported values are:

- `SPIN`, `'SPIN'`, `{-1, 1}`
- `BINARY`, `'BINARY'`, `{0, 1}`

### **constraints**

Constraints that together constitute the constraint satisfaction problem. Valid solutions satisfy all of the constraints.

**Type** list[*Constraint*]

### **variables**

Variables of the constraint satisfaction problem as a dict, where keys are the variables and values a list of all of constraints associated with the variable.

**Type** dict[variable, list[*Constraint*]]

### **vartype**

Enumeration of valid variable types. Supported values are `SPIN` or `BINARY`. If *vartype* is `SPIN`, variables can be assigned -1 or 1; if `BINARY`, variables can be assigned 0 or 1.

**Type** `dimod.Vartype`

## Example

This example creates a binary-valued constraint satisfaction problem, adds two constraints,  $a = b$  and  $b \neq c$ , and tests  $a, b, c = 1, 1, 0$ .

```
>>> import operator
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem('BINARY')
>>> csp.add_constraint(operator.eq, ['a', 'b'])
>>> csp.add_constraint(operator.ne, ['b', 'c'])
>>> csp.check({'a': 1, 'b': 1, 'c': 0})
True
```

## Methods

### Adding variables and constraints

<code>ConstraintSatisfactionProblem.add_constraint(...)</code>	Add a constraint.
<code>ConstraintSatisfactionProblem.add_variable(v)</code>	Add a variable.

**dwavebinarycsp.ConstraintSatisfactionProblem.add\_constraint**

ConstraintSatisfactionProblem.**add\_constraint** (*constraint*, *variables=()*)

Add a constraint.

**Parameters**

- **constraint** (function/iterable/*Constraint*) – Constraint definition in one of the supported formats:
  1. Function, with input arguments matching the order and *vartype* type of the *variables* argument, that evaluates True when the constraint is satisfied.
  2. List explicitly specifying each allowed configuration as a tuple.
  3. *Constraint* object built either explicitly or by `dwavebinarycsp.factories`.
- **variables** (*iterable*) – Variables associated with the constraint. Not required when *constraint* is a *Constraint* object.

**Examples**

This example defines a function that evaluates True when the constraint is satisfied. The function's input arguments match the order and type of the *variables* argument.

```
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> def all_equal(a, b, c): # works for both dwavebinarycsp.BINARY and_
↳dwavebinarycsp.SPIN
...     return (a == b) and (b == c)
>>> csp.add_constraint(all_equal, ['a', 'b', 'c'])
>>> csp.check({'a': 0, 'b': 0, 'c': 0})
True
>>> csp.check({'a': 0, 'b': 0, 'c': 1})
False
```

This example explicitly lists allowed configurations.

```
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.SPIN)
>>> eq_configurations = {(-1, -1), (1, 1)}
>>> csp.add_constraint(eq_configurations, ['v0', 'v1'])
>>> csp.check({'v0': -1, 'v1': +1})
False
>>> csp.check({'v0': -1, 'v1': -1})
True
```

This example uses a *Constraint* object built by `dwavebinarycsp.factories`.

```
>>> import dwavebinarycsp.factories.constraint.gates as gates
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(gates.and_gate(['a', 'b', 'c'])) # add an AND gate
>>> csp.add_constraint(gates.xor_gate(['a', 'c', 'd'])) # add an XOR gate
>>> csp.check({'a': 1, 'b': 0, 'c': 0, 'd': 1})
True
```

## dwavebinarycsp.ConstraintSatisfactionProblem.add\_variable

ConstraintSatisfactionProblem.**add\_variable**(*v*)

Add a variable.

**Parameters** *v* (*variable*) – Variable in the constraint satisfaction problem. May be of any type that can be a dict key.

### Examples

This example adds two variables, one of which is already used in a constraint of the constraint satisfaction problem.

```

>>> import operator
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.SPIN)
>>> csp.add_constraint(operator.eq, ['a', 'b'])
>>> csp.add_variable('a') # does nothing, already added as part of the constraint
>>> csp.add_variable('c')
>>> csp.check({'a': -1, 'b': -1, 'c': 1})
True
>>> csp.check({'a': -1, 'b': -1, 'c': -1})
True

```

## Satisfiability

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*ConstraintSatisfactionProblem*.  
*check*(*solution*)

---

Check that a solution satisfies all of the constraints.

## dwavebinarycsp.ConstraintSatisfactionProblem.check

ConstraintSatisfactionProblem.**check**(*solution*)

Check that a solution satisfies all of the constraints.

**Parameters** *solution* (*container*) – An assignment of values for the variables in the constraint satisfaction problem.

**Returns** True if the solution satisfies all of the constraints; False otherwise.

**Return type** bool

### Examples

This example creates a binary-valued constraint satisfaction problem, adds two logic gates implementing Boolean constraints,  $c = a \wedge b$  and  $d = a \oplus c$ , and verifies that the combined problem is satisfied for a given assignment.

```

>>> import dwavebinarycsp.factories.constraint.gates as gates
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(gates.and_gate(['a', 'b', 'c'])) # add an AND gate
>>> csp.add_constraint(gates.xor_gate(['a', 'c', 'd'])) # add an XOR gate
>>> csp.check({'a': 1, 'b': 0, 'c': 0, 'd': 1})
True

```

## Transformations

---

<code>ConstraintSatisfactionProblem. fix_variable(v,...)</code>	Fix the value of a variable and remove it from the constraint satisfaction problem.
---	---

---

### `dwavebinarycsp.ConstraintSatisfactionProblem.fix_variable`

`ConstraintSatisfactionProblem.fix_variable(v, value)`

Fix the value of a variable and remove it from the constraint satisfaction problem.

#### Parameters

- **v** (*variable*) – Variable to be fixed in the constraint satisfaction problem.
- **value** (*int*) – Value assigned to the variable. Values must match the *vartype* of the constraint satisfaction problem.

#### Examples

This example creates a spin-valued constraint satisfaction problem, adds two constraints,  $a = b$  and  $b \neq c$ , and fixes variable `b` to `+1`.

```
>>> import operator
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.SPIN)
>>> csp.add_constraint(operator.eq, ['a', 'b'])
>>> csp.add_constraint(operator.ne, ['b', 'c'])
>>> csp.check({'a': +1, 'b': +1, 'c': -1})
True
>>> csp.check({'a': -1, 'b': -1, 'c': +1})
True
>>> csp.fix_variable('b', +1)
>>> csp.check({'a': +1, 'b': +1, 'c': -1}) # 'b' is ignored
True
>>> csp.check({'a': -1, 'b': -1, 'c': +1})
False
>>> csp.check({'a': +1, 'c': -1})
True
>>> csp.check({'a': -1, 'c': +1})
False
```

## 1.2.2 Converting to a Binary Quadratic Model

Constraint satisfaction problems can be converted to binary quadratic models to be solved on samplers such as the D-Wave system.

### Compilers

Compilers accept a constraint satisfaction problem and return a `dimod.BinaryQuadraticModel`.

---

<code>stitch(csp[, min_classical_gap, max_graph_size])</code>	Build a binary quadratic model with minimal energy levels at solutions to the specified constraint satisfaction problem.
---	--

---

## dwavebinarycsp.stitch

**stitch** (*csp*, *min\_classical\_gap*=2.0, *max\_graph\_size*=8)

Build a binary quadratic model with minimal energy levels at solutions to the specified constraint satisfaction problem.

### Parameters

- **csp** (*ConstraintSatisfactionProblem*) – Constraint satisfaction problem.
- **min\_classical\_gap** (*float*, *optional*, *default*=2.0) – Minimum energy gap from ground. Each constraint violated by the solution increases the energy level of the binary quadratic model by at least this much relative to ground energy.
- **max\_graph\_size** (*int*, *optional*, *default*=8) – Maximum number of variables in the binary quadratic model that can be used to represent a single constraint.

**Returns** `BinaryQuadraticModel`

### Notes

For a *min\_classical\_gap* > 2 or constraints with more than two variables, requires access to factories from the `penaltymodel` ecosystem to construct the binary quadratic model.

### Examples

This example creates a binary-valued constraint satisfaction problem with two constraints,  $a = b$  and  $b \neq c$ , and builds a binary quadratic model such that each constraint violation by a solution adds the default minimum energy gap.

```
>>> import operator
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(operator.eq, ['a', 'b']) # a == b
>>> csp.add_constraint(operator.ne, ['b', 'c']) # b != c
>>> bqm = dwavebinarycsp.stitch(csp)
```

Variable assignments that satisfy the CSP above, violate one, then two constraints, produce energy increases of the default minimum classical gap:

```
>>> bqm.energy({'a': 0, 'b': 0, 'c': 1}) # doctest: +SKIP
-2.0
>>> bqm.energy({'a': 0, 'b': 0, 'c': 0}) # doctest: +SKIP
0.0
>>> bqm.energy({'a': 1, 'b': 0, 'c': 0}) # doctest: +SKIP
2.0
```

This example creates a binary-valued constraint satisfaction problem with two constraints,  $a = b$  and  $b \neq c$ , and builds a binary quadratic model with a minimum energy gap of 4. Note that in this case the conversion to binary quadratic model adds two ancillary variables that must be minimized over when solving.

```
>>> import operator
>>> import itertools
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(operator.eq, ['a', 'b']) # a == b
>>> csp.add_constraint(operator.ne, ['b', 'c']) # b != c
>>> bqm = dwavebinarycsp.stitch(csp, min_classical_gap=4.0)
>>> list(bqm) # doctest: +SKIP
['a', 'aux1', 'aux0', 'b', 'c']
```

Variable assignments that satisfy the CSP above, violate one, then two constraints, produce energy increases of the specified minimum classical gap:

```
>>> min([bqm.energy({'a': 0, 'b': 0, 'c': 1, 'aux0': aux0, 'aux1': aux1}) for
... aux0, aux1 in list(itertools.product([0, 1], repeat=2))]) # doctest: +SKIP
-6.0
>>> min([bqm.energy({'a': 0, 'b': 0, 'c': 0, 'aux0': aux0, 'aux1': aux1}) for
... aux0, aux1 in list(itertools.product([0, 1], repeat=2))]) # doctest: +SKIP
-2.0
>>> min([bqm.energy({'a': 1, 'b': 0, 'c': 0, 'aux0': aux0, 'aux1': aux1}) for
... aux0, aux1 in list(itertools.product([0, 1], repeat=2))]) # doctest: +SKIP
2.0
```

This example finds for the previous example the minimum graph size.

```
>>> import operator
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(operator.eq, ['a', 'b']) # a == b
>>> csp.add_constraint(operator.ne, ['b', 'c']) # b != c
>>> for n in range(8, 1, -1):
...     try:
...         bqm = dwavebinarycsp.stitch(csp, min_classical_gap=4.0, max_graph_
↪size=n)
...         except dwavebinarycsp.exceptions.ImpossibleBQM:
...             print(n+1)
...
3
```

## 1.2.3 Other CSP Formats

### DIMACS

The DIMACS format is used to encode boolean satisfiability problems in conjunctive normal form.

### CNF

---

<code>load_cnf(fp)</code>	Load a constraint satisfaction problem from a .cnf file.
---------------------------	--

### dwavebinarycsp.io.cnf.load\_cnf

#### `load_cnf(fp)`

Load a constraint satisfaction problem from a .cnf file.

**Parameters** `fp` (*file, optional*) – *.write()*-supporting file object DIMACS CNF formatted



file.

**Returns** *ConstraintSatisfactionProblem* a binary-valued SAT problem.

### Examples

```
>>> import dwavebinarycsp as dbcsp
...
>>> with open('test.cnf', 'r') as fp: # doctest: +SKIP
...     csp = dbcsp.cnf.load_cnf(fp)
```

## 1.2.4 Reducing Constraints

Constraints can sometimes be reduced into several smaller constraints.

### Functions

---

<i>irreducible_components</i> (constraint)	Determine the sets of variables that are irreducible.
--	---

---

#### `dwavebinarycsp.irreducible_components`

**irreducible\_components** (*constraint*)

Determine the sets of variables that are irreducible.

Let  $V(C)$  denote the variables of constraint  $C$ . For a configuration  $x$ , let  $x|A$  denote the restriction of the configuration to the variables of  $A$ . Constraint  $C$  is reducible if there is a partition of  $V(C)$  into nonempty subsets  $A$  and  $B$ , and two constraints  $C_A$  and  $C_B$ , with  $V(C_A) = A$  and  $C_B \vee V(C_B) = B$ , such that a configuration  $x$  is feasible in  $C$  if and only if  $x|A$  is feasible in  $C_A$  and  $x|B$  is feasible in  $C_B$ . A constraint is irreducible if it is not reducible.

**Parameters** **constraint** (*Constraint*) – Constraint to attempt to reduce.

**Returns** List of tuples in which each tuple is a set of variables that is irreducible.

**Return type** `list[tuple]`

### Examples

This example reduces a constraint, created by specifying its valid configurations, to two constraints. The original constraint, that valid configurations for  $a, b, c$  are  $0, 0, 1$  and  $1, 1, 1$ , can be represented by two reduced constraints, for example,  $(c=1) \ \& \ (a=b)$ . For comparison, an attempt to reduce a constraint representing an AND gate fails to find a valid reduction.

```
>>> const = dwavebinarycsp.Constraint.from_configurations([(0, 0, 1), (1, 1, 1)],
...                                                       ['a', 'b', 'c'],
...                                                       dwavebinarycsp.BINARY)
>>> dwavebinarycsp.irreducible_components(const)
[('c',), ('a', 'b')]
>>> const_and = dwavebinarycsp.Constraint.from_configurations([(0, 0, 0), (0, 1,
... 0), (1, 0, 0), (1, 1, 1)],
...                                                       ['a', 'b', 'c'],
...                                                       dwavebinarycsp.BINARY)
```

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```
>>> dwavebinarycsp.irreducible_components(const_and)
[('a', 'b', 'c')]
```

## 1.2.5 Defining Constraints

Solutions to a constraint satisfaction problem must satisfy certain conditions, the constraints of the problem, such as equality and inequality constraints. The `Constraint` class defines constraints and provides functionality to assist in constraint definition, such as verifying whether a candidate solution satisfies a constraint.

### Class

**class Constraint** (*func, configurations, variables, vartype, name=None*)

A constraint.

**variables**

Variables associated with the constraint.

**Type** tuple

**func**

Function that returns True for configurations of variables that satisfy the constraint. Inputs to the function are ordered by *variables*.

**Type** function

**configurations**

Valid configurations of the variables. Each configuration is a tuple of variable assignments ordered by *variables*.

**Type** frozenset[tuple]

**vartype**

Variable type for the constraint. Accepted input values:

- `SPIN`, 'SPIN', {-1, 1}
- `BINARY`, 'BINARY', {0, 1}

**Type** `dimod.Vartype`

**name**

Name for the constraint. If not provided on construction, defaults to 'Constraint'.

**Type** str

### Examples

This example defines a constraint, named “plus1”, based on a function that is True for  $(y1, y0) = (x1, x0) + 1$  on binary variables, and demonstrates some of the constraint’s functionality.

```
>>> def plus_one(y1, y0, x1, x0): # y=x++ for two bit binary numbers
...     return (y1, y0, x1, x0) in [(0, 1, 0, 0), (1, 0, 0, 1), (1, 1, 1, 0)]
...
>>> const = dwavebinarycsp.Constraint.from_func(
...     plus_one,
```

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```

...         ['out1', 'out0', 'in1', 'in0'],
...         dwavebinarycsp.BINARY,
...         name='plus1')
>>> print(const.name)    # Check constraint defined as intended
plus1
>>> len(const)
4
>>> in0, in1, out0, out1 = 0, 0, 1, 0
>>> const.func(out1, out0, in1, in0)    # Order matches variables
True

```

This example defines a constraint based on specified valid configurations that represents an AND gate for spin variables, and demonstrates some of the constraint's functionality.

```

>>> const = dwavebinarycsp.Constraint.from_configurations(
...     [(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (1, 1, 1)],
...     ['y', 'x1', 'x2'],
...     dwavebinarycsp.SPIN)
>>> print(const.name)    # Check constraint defined as intended
Constraint
>>> isinstance(const, dwavebinarycsp.core.constraint.Constraint)
True
>>> (-1, 1, -1) in const.configurations    # Order matches variables: y,x1,x2
True

```

## Methods

### Construction

---

<code>Constraint.from_configurations(...[, name])</code>	Construct a constraint from valid configurations.
--	---

---

<code>Constraint.from_func(func, variables, vartype)</code>	Construct a constraint from a validation function.
---	--

---

### `dwavebinarycsp.Constraint.from_configurations`

**classmethod** `Constraint.from_configurations` (*configurations*, *variables*, *vartype*, *name=None*)

Construct a constraint from valid configurations.

#### Parameters

- **configurations** (*iterable[tuple]*) – Valid configurations of the variables. Each configuration is a tuple of variable assignments ordered by *variables*.
- **variables** (*iterable*) – Iterable of variable labels.
- **vartype** (*Vartype/str/set*) – Variable type for the constraint. Accepted input values:
  - `SPIN`, `'SPIN'`, `{-1, 1}`
  - `BINARY`, `'BINARY'`, `{0, 1}`
- **name** (*string, optional, default='Constraint'*) – Name for the constraint.

## Examples

This example creates a constraint that variables  $a$  and  $b$  are not equal.

```
>>> const = dwavebinarycsp.Constraint.from_configurations([(0, 1), (1, 0)],
...               ['a', 'b'], dwavebinarycsp.BINARY)
>>> print(const.name)
Constraint
>>> (0, 0) in const.configurations # Order matches variables: a,b
False
```

This example creates a constraint based on specified valid configurations that represents an OR gate for spin variables.

```
>>> const = dwavebinarycsp.Constraint.from_configurations(
...     [(-1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)],
...     ['y', 'x1', 'x2'],
...     dwavebinarycsp.SPIN, name='or_spin')
>>> print(const.name)
or_spin
>>> (1, 1, -1) in const.configurations # Order matches variables: y,x1,x2
True
```

## dwavebinarycsp.Constraint.from\_func

**classmethod** `Constraint.from_func` (*func*, *variables*, *vartype*, *name=None*)

Construct a constraint from a validation function.

### Parameters

- **func** (*function*) – Function that evaluates True when the variables satisfy the constraint.
- **variables** (*iterable*) – Iterable of variable labels.
- **vartype** (*Vartype/str/set*) – Variable type for the constraint. Accepted input values:
  - `SPIN`, `'SPIN'`, `{-1, 1}`
  - `BINARY`, `'BINARY'`, `{0, 1}`
- **name** (*string, optional, default='Constraint'*) – Name for the constraint.

## Examples

This example creates a constraint that binary variables  $a$  and  $b$  are not equal.

```
>>> import operator
>>> const = dwavebinarycsp.Constraint.from_func(operator.ne, ['a', 'b'], 'BINARY')
>>> print(const.name)
Constraint
>>> (0, 1) in const.configurations
True
```

This example creates a constraint that  $out = NOT(x)$  for spin variables.

```

>>> def not_(y, x): # y=NOT(x) for spin variables
...     return (y == -x)
...
>>> const = dwavebinarycsp.Constraint.from_func(
...     not_,
...     ['out', 'in'],
...     {1, -1},
...     name='not_spin')
>>> print(const.name)
not_spin
>>> (1, -1) in const.configurations
True

```

## Satisfiability

---

<code>Constraint.check(solution)</code>	Check that a solution satisfies the constraint.
---	---

---

### `dwavebinarycsp.Constraint.check`

`Constraint.check(solution)`

Check that a solution satisfies the constraint.

**Parameters** `solution` (*container*) – An assignment for the variables in the constraint.

**Returns** True if the solution satisfies the constraint; otherwise False.

**Return type** `bool`

### Examples

This example creates a constraint that  $a \neq b$  on binary variables and tests it for two candidate solutions, with additional unconstrained variable `c`.

```

>>> const = dwavebinarycsp.Constraint.from_configurations([(0, 1), (1, 0)],
...     ['a', 'b'], dwavebinarycsp.BINARY)
>>> solution = {'a': 1, 'b': 1, 'c': 0}
>>> const.check(solution)
False
>>> solution = {'a': 1, 'b': 0, 'c': 0}
>>> const.check(solution)
True

```

## Transformations

---

<code>Constraint.fix_variable(v, value)</code>	Fix the value of a variable and remove it from the constraint.
<code>Constraint.flip_variable(v)</code>	Flip a variable in the constraint.

---

**dwavebinarycsp.Constraint.fix\_variable**

`Constraint.fix_variable(v, value)`

Fix the value of a variable and remove it from the constraint.

**Parameters**

- **v** (*variable*) – Variable in the constraint to be set to a constant value.
- **val** (*int*) – Value assigned to the variable. Values must match the `Vartype` of the constraint.

**Examples**

This example creates a constraint that  $a \neq b$  on binary variables, fixes variable `a` to 0, and tests two candidate solutions.

```
>>> const = dwavebinarycsp.Constraint.from_func(operator.ne,
...         ['a', 'b'], dwavebinarycsp.BINARY)
>>> const.fix_variable('a', 0)
>>> const.check({'b': 1})
True
>>> const.check({'b': 0})
False
```

**dwavebinarycsp.Constraint.flip\_variable**

`Constraint.flip_variable(v)`

Flip a variable in the constraint.

- Parameters** **v** (*variable*) – Variable in the constraint to take the complementary value of its construction value.

**Examples**

This example creates a constraint that  $a = b$  on binary variables and flips variable `a`.

```
>>> const = dwavebinarycsp.Constraint.from_func(operator.eq,
...         ['a', 'b'], dwavebinarycsp.BINARY)
>>> const.check({'a': 0, 'b': 0})
True
>>> const.flip_variable('a')
>>> const.check({'a': 1, 'b': 0})
True
>>> const.check({'a': 0, 'b': 0})
False
```

**Copies and projections**

---

`Constraint.copy()`

Create a copy.

Continued on next page

Table 11 – continued from previous page

<code>Constraint.projection(variables)</code>	Create a new constraint that is the projection onto a subset of the variables.
---	--

### `dwavebinarycsp.Constraint.copy`

`Constraint.copy()`  
Create a copy.

#### Examples

This example copies constraint  $a \neq b$  and tests a solution on the copied constraint.

```
>>> import operator
>>> const = dwavebinarycsp.Constraint.from_func(operator.ne,
...      ['a', 'b'], 'BINARY')
>>> const2 = const.copy()
>>> const2 is const
False
>>> const2.check({'a': 1, 'b': 1})
False
```

### `dwavebinarycsp.Constraint.projection`

`Constraint.projection(variables)`  
Create a new constraint that is the projection onto a subset of the variables.

**Parameters** `variables` (*iterable*) – Subset of the constraint’s variables.

**Returns** A new constraint over a subset of the variables.

**Return type** `Constraint`

#### Examples

```
>>> const = dwavebinarycsp.Constraint.from_configurations([(0, 0), (0, 1)],
...      ['a', 'b'],
...      dwavebinarycsp.BINARY)
>>> proj = const.projection(['a'])
>>> proj.variables
('a',)
>>> proj.configurations
frozenset({(0,)})
```

## 1.2.6 Factories

`dwavebinarycsp` currently provides factories for constraints representing Boolean gates and satisfiability problems and CSPs for circuits and satisfiability problems.

## Constraints

### Gates

<code>gates.and_gate(variables[, vartype, name])</code>	AND gate.
<code>gates.or_gate(variables[, vartype, name])</code>	OR gate.
<code>gates.xor_gate(variables[, vartype, name])</code>	XOR gate.
<code>gates.halfadder_gate(variables[, vartype, name])</code>	Half adder.
<code>gates.fulladder_gate(variables[, vartype, name])</code>	Full adder.

### dwavebinarycsp.factories.constraint.gates.and\_gate

**and\_gate** (*variables*, *vartype*=<Vartype.BINARY: frozenset({0, 1})>, *name*='AND')

AND gate.

#### Parameters

- **variables** (*list*) – Variable labels for the and gate as [*in1*, *in2*, *out*], where *in1*, *in2* are inputs and *out* the gate’s output.
- **vartype** (*Vartype*, *optional*, *default*='BINARY') – Variable type. Accepted input values:
  - Vartype.SPIN, ‘SPIN’, {-1, 1}
  - Vartype.BINARY, ‘BINARY’, {0, 1}
- **name** (*str*, *optional*, *default*='AND') – Name for the constraint.

**Returns** Constraint that is satisfied when its variables are assigned values that match the valid states of an AND gate.

**Return type** Constraint(*Constraint*)

#### Examples

```
>>> import dwavebinarycsp.factories.constraint.gates as gates
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(gates.and_gate(['a', 'b', 'c'], name='AND1'))
>>> csp.check({'a': 1, 'b': 0, 'c': 0})
True
```

### dwavebinarycsp.factories.constraint.gates.or\_gate

**or\_gate** (*variables*, *vartype*=<Vartype.BINARY: frozenset({0, 1})>, *name*='OR')

OR gate.

#### Parameters

- **variables** (*list*) – Variable labels for the and gate as [*in1*, *in2*, *out*], where *in1*, *in2* are inputs and *out* the gate’s output.



- **vartype** (*Vartype*, *optional*, *default='BINARY'*) – Variable type. Accepted input values:
  - Vartype.SPIN, 'SPIN', {-1, 1}
  - Vartype.BINARY, 'BINARY', {0, 1}
- **name** (*str*, *optional*, *default='OR'*) – Name for the constraint.

**Returns** Constraint that is satisfied when its variables are assigned values that match the valid states of an OR gate.

**Return type** Constraint(*Constraint*)

### Examples

```
>>> import dwavebinarycsp.factories.constraint.gates as gates
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.SPIN)
>>> csp.add_constraint(gates.or_gate(['x', 'y', 'z'], {-1,1}, name='OR1'))
>>> csp.check({'x': 1, 'y': -1, 'z': 1})
True
```

### dwavebinarycsp.factories.constraint.gates.xor\_gate

**xor\_gate** (*variables*, *vartype=<Vartype.BINARY: frozenset({0, 1})>*, *name='XOR'*)  
XOR gate.

#### Parameters

- **variables** (*list*) – Variable labels for the and gate as [*in1*, *in2*, *out*], where *in1*, *in2* are inputs and *out* the gate's output.
- **vartype** (*Vartype*, *optional*, *default='BINARY'*) – Variable type. Accepted input values:
  - Vartype.SPIN, 'SPIN', {-1, 1}
  - Vartype.BINARY, 'BINARY', {0, 1}
- **name** (*str*, *optional*, *default='XOR'*) – Name for the constraint.

**Returns** Constraint that is satisfied when its variables are assigned values that match the valid states of an XOR gate.

**Return type** Constraint(*Constraint*)

### Examples

```
>>> import dwavebinarycsp.factories.constraint.gates as gates
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(gates.xor_gate(['x', 'y', 'z'], name='XOR1'))
>>> csp.check({'x': 1, 'y': 1, 'z': 1})
False
```

### dwavebinarycsp.factories.constraint.gates.halfadder\_gate

**halfadder\_gate** (*variables*, *vartype*=<Vartype.BINARY: frozenset({0, 1})>, *name*='HALF\_ADDER')  
 Half adder.

#### Parameters

- **variables** (*list*) – Variable labels for the and gate as [*in1*, *in2*, *sum*, *carry*], where *in1*, *in2* are inputs to be added and *sum* and ‘carry’ the resultant outputs.
- **vartype** (*Vartype*, *optional*, *default*='BINARY') – Variable type. Accepted input values:
  - Vartype.SPIN, ‘SPIN’, {-1, 1}
  - Vartype.BINARY, ‘BINARY’, {0, 1}
- **name** (*str*, *optional*, *default*='HALF\_ADDER') – Name for the constraint.

**Returns** Constraint that is satisfied when its variables are assigned values that match the valid states of a Boolean half adder.

**Return type** Constraint(*Constraint*)

#### Examples

```
>>> import dwavebinarycsp.factories.constraint.gates as gates
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(gates.halfadder_gate(['a', 'b', 'total', 'carry'], name=
↳ 'HAL'))
>>> csp.check({'a': 1, 'b': 1, 'total': 0, 'carry': 1})
True
```

### dwavebinarycsp.factories.constraint.gates.fulladder\_gate

**fulladder\_gate** (*variables*, *vartype*=<Vartype.BINARY: frozenset({0, 1})>, *name*='FULL\_ADDER')  
 Full adder.

#### Parameters

- **variables** (*list*) – Variable labels for the and gate as [*in1*, *in2*, *in3*, *sum*, *carry*], where *in1*, *in2*, *in3* are inputs to be added and *sum* and ‘carry’ the resultant outputs.
- **vartype** (*Vartype*, *optional*, *default*='BINARY') – Variable type. Accepted input values:
  - Vartype.SPIN, ‘SPIN’, {-1, 1}
  - Vartype.BINARY, ‘BINARY’, {0, 1}
- **name** (*str*, *optional*, *default*='FULL\_ADDER') – Name for the constraint.

**Returns** Constraint that is satisfied when its variables are assigned values that match the valid states of a Boolean full adder.

**Return type** Constraint(*Constraint*)

## Examples

```
>>> import dwavebinarycsp.factories.constraint.gates as gates
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(gates.fulladder_gate(['a', 'b', 'c_in', 'total', 'c_out'],
↳name='FA1'))
>>> csp.check({'a': 1, 'b': 0, 'c_in': 1, 'total': 0, 'c_out': 1})
True
```

## Satisfiability Problems

---

<code>sat.sat2in4(pos[, neg, vartype, name])</code>	Two-in-four (2-in-4) satisfiability.
---	--------------------------------------

---

### `dwavebinarycsp.factories.constraint.sat.sat2in4`

**sat2in4** (*pos*, *neg*=(), *vartype*=<*Vartype.BINARY: frozenset({0, 1})*>, *name*='2-in-4')

Two-in-four (2-in-4) satisfiability.

#### Parameters

- **pos** (*iterable*) – Variable labels, as an iterable, for non-negated variables of the constraint. Exactly four variables are specified by *pos* and *neg* together.
- **neg** (*tuple*) – Variable labels, as an iterable, for negated variables of the constraint. Exactly four variables are specified by *pos* and *neg* together.
- **vartype** (*Vartype*, *optional*, *default*='BINARY') – Variable type. Accepted input values:
  - *Vartype.SPIN*, 'SPIN', {-1, 1}
  - *Vartype.BINARY*, 'BINARY', {0, 1}
- **name** (*str*, *optional*, *default*='2-in-4') – Name for the constraint.

**Returns** Constraint that is satisfied when its variables are assigned values that satisfy a two-in-four satisfiability problem.

**Return type** Constraint(*Constraint*)

## Examples

```
>>> import dwavebinarycsp.factories.constraint.sat as sat
>>> csp = dwavebinarycsp.ConstraintSatisfactionProblem(dwavebinarycsp.BINARY)
>>> csp.add_constraint(sat.sat2in4(['w', 'x', 'y', 'z'], vartype='BINARY', name=
↳'sat1'))
>>> csp.check({'w': 1, 'x': 1, 'y': 0, 'z': 0})
True
```

## CSPs

<code>circuits.multiplication_circuit(nbit[, vartype])</code>	Multiplication circuit constraint satisfaction problem.
<code>sat.random_2in4sat(num_variables, num_clauses)</code>	Random two-in-four (2-in-4) constraint satisfaction problem.
<code>sat.random_xorsat(num_variables, num_clauses)</code>	Random XOR constraint satisfaction problem.

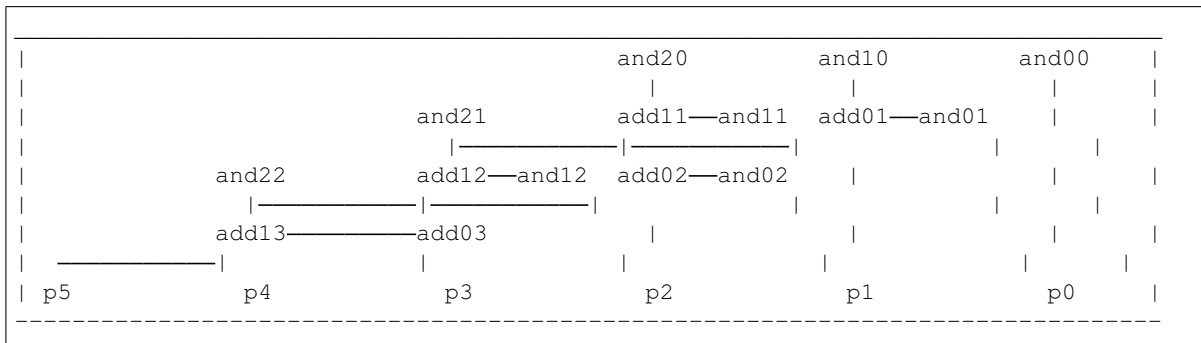
### dwavebinarycsp.factories.csp.circuits.multiplication\_circuit

**multiplication\_circuit** (*nbit*, *vartype*=<*Vartype*.*BINARY*: frozenset({0, 1})>)

Multiplication circuit constraint satisfaction problem.

A constraint satisfaction problem that represents the binary multiplication  $ab = p$ , where the multiplicands are binary variables of length *nbit*; for example,  $2^m a_{nbit} + \dots + 4a_2 + 2a_1 + a_0$ .

The square below shows a graphic representation of the circuit:



#### Parameters

- **nbit** (*int*) – Number of bits in the multiplicands.
- **vartype** (*Vartype*, *optional*, *default*='BINARY') – Variable type. Accepted input values:
  - *Vartype*.SPIN, 'SPIN', {-1, 1}
  - *Vartype*.BINARY, 'BINARY', {0, 1}

**Returns** CSP that is satisfied when variables *a*, *b*, *p* are assigned values that correctly solve binary multiplication  $ab = p$ .

**Return type** CSP (*ConstraintSatisfactionProblem*)

#### Examples

This example creates a multiplication circuit CSP that multiplies two 3-bit numbers, which is then formulated as a binary quadratic model (BQM). It fixes the multiplicands as  $a = 5$ ,  $b = 3$  (101 and 011) and uses a simulated annealing sampler to find the product,  $p = 15$  (001111).

```
>>> from dwavebinarycsp.factories.csp.circuits import multiplication_circuit
>>> import neal
>>> csp = multiplication_circuit(3)
>>> bqm = dwavebinarycsp.stitch(csp)
>>> bqm.fix_variable('a0', 1); bqm.fix_variable('a1', 0); bqm.fix_variable('a2', 1)
↪1)
```

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```

>>> bqm.fix_variable('b0', 1); bqm.fix_variable('b1', 1); bqm.fix_variable('b2', 1)
↳0)
>>> sampler = neal.SimulatedAnnealingSampler()
>>> response = sampler.sample(bqm)
>>> p = next(response.samples(n=1, sorted_by='energy'))
>>> print(p['p5'], p['p4'], p['p3'], p['p2'], p['p1'], p['p0'])      # doctest:↳
↳+SKIP
0 0 1 1 1 1

```

## dwavebinarycsp.factories.csp.sat.random\_2in4sat

**random\_2in4sat** (*num\_variables*, *num\_clauses*, *vartype*=<*Vartype*.*BINARY*: frozenset({0, 1})>, *satisfiable*=True)

Random two-in-four (2-in-4) constraint satisfaction problem.

### Parameters

- **num\_variables** (*integer*) – Number of variables (at least four).
- **num\_clauses** (*integer*) – Number of constraints that together constitute the constraint satisfaction problem.
- **vartype** (*Vartype*, *optional*, *default*='BINARY') – Variable type. Accepted input values:
  - *Vartype*.SPIN, 'SPIN', {-1, 1}
  - *Vartype*.BINARY, 'BINARY', {0, 1}
- **satisfiable** (*bool*, *optional*, *default*=True) – True if the CSP can be satisfied.

**Returns** CSP that is satisfied when its variables are assigned values that satisfy a two-in-four satisfiability problem.

**Return type** CSP (*ConstraintSatisfactionProblem*)

### Examples

This example creates a CSP with 6 variables and two random constraints and checks whether a particular assignment of variables satisfies it.

```

>>> import dwavebinarycsp.factories as sat
>>> csp = sat.random_2in4sat(6, 2)
>>> csp.constraints      # doctest: +SKIP
[Constraint.from_configurations(frozenset({(1, 0, 1, 0), (1, 0, 0, 1), (1, 1, 1, 1),
↳1), (0, 1, 1, 0), (0, 0, 0, 0),
(0, 1, 0, 1)}), (2, 4, 0, 1), Vartype.BINARY, name='2-in-4'),
Constraint.from_configurations(frozenset({(1, 0, 1, 1), (1, 1, 0, 1), (1, 1, 1, 1),
↳0), (0, 0, 0, 1),
(0, 1, 0, 0), (0, 0, 1, 0)}), (1, 2, 4, 5), Vartype.BINARY, name='2-in-4')]
>>> csp.check({0: 1, 1: 0, 2: 1, 3: 1, 4: 0, 5: 0})      # doctest: +SKIP
True

```

**dwavebinarycsp.factories.csp.sat.random\_xorsat**

**random\_xorsat** (*num\_variables*, *num\_clauses*, *vartype*=<*Vartype.BINARY*: frozenset({0, 1})>, *satisfiable*=True)  
Random XOR constraint satisfaction problem.

**Parameters**

- **num\_variables** (*integer*) – Number of variables (at least three).
- **num\_clauses** (*integer*) – Number of constraints that together constitute the constraint satisfaction problem.
- **vartype** (*Vartype*, *optional*, *default*='BINARY') – Variable type. Accepted input values:
  - *Vartype*.SPIN, 'SPIN', {-1, 1}
  - *Vartype*.BINARY, 'BINARY', {0, 1}
- **satisfiable** (*bool*, *optional*, *default*=True) – True if the CSP can be satisfied.

**Returns** CSP that is satisfied when its variables are assigned values that satisfy a XOR satisfiability problem.

**Return type** CSP (*ConstraintSatisfactionProblem*)

**Examples**

This example creates a CSP with 5 variables and two random constraints and checks whether a particular assignment of variables satisfies it.

```
>>> import dwavebinarycsp.factories as sat
>>> csp = sat.random_xorsat(5, 2)
>>> csp.constraints      # doctest: +SKIP
[Constraint.from_configurations(frozenset({(1, 0, 0), (1, 1, 1), (0, 1, 0), (0, 0,
↪ 1)}), (4, 3, 0),
  Vartype.BINARY, name='XOR (0 flipped)'),
  Constraint.from_configurations(frozenset({(1, 1, 0), (0, 1, 1), (0, 0, 0), (1, 0,
↪ 1)}), (2, 0, 4),
  Vartype.BINARY, name='XOR (2 flipped) (0 flipped)')]
>>> csp.check({0: 1, 1: 0, 2: 0, 3: 1, 4: 1})      # doctest: +SKIP
True
```

## 1.3 Bibliography

## 1.4 Installation

To install:

```
pip install dwavebinarycsp
```

To build from source:

```
pip install -r requirements.txt
python setup.py install
```

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## CHAPTER 2

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